Thin-Walled Structures 74 (2014) 213–221

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Buckling and deformation of Hollomon's power-law tubes

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ARTICLE INFO

Article history:
Received 14 April 2013
Received in revised form
1 October 2013
Accepted 1 October 2013
Available online 31 October 2013

Keywords:
High strength material
Hollomon power-law
Buckling and post-buckling
Deformation of cylindrical tubes
Deformation of circular rings
Differential system of equations

ABSTRACT

A long, thin, inextensible cylindrical tube made of Hollomon's power-law material acted upon by a uniform normal pressure is considered. The nonlinear boundary value problem that governs the equilibrium states of such a tube is formulated as a differential system of equations. Perturbation solutions are obtained for the cases of small pressure values in the neighborhood of the critical buckling pressures. Numerical solutions based on a special initial value problem Matlab solver, Newton's and shooting methods are obtained. The results show that a high strength material tube deforms similarly to an elastic material tube for values of strain hardening exponent.

1. Introduction

The development of high-strength material made significant contributions in oil and gas industries, as well as automobile, aircraft, ships and submarine industries. The development focuses on the strength properties without reducing the toughness of the materials and requires understanding of the strength capabilities of the material both for small (buckling) and for large (post-buckling) deformations due to small and large forces, respectively. The purpose of this paper is to determine the buckling loads as well as the buckling and post-buckling shapes for a circular, long, inextensible tube made of high strength material subject to a normal uniform external pressure. This study should be useful in the design methodologies for pipeline, bridges, buildings, aircraft, submarine and other structures made of high strength material.

Buckling and post-buckling of circular tubes made of linear (elastic) material subject to uniform and non-uniform external pressure are examined analytically, and numerically in many previous works, for example, see [1–5] and the reference therein. Several mathematical formulations for the equations that describe the equilibrium states of such tubes are given in these references. Among other things, it is determined that the buckling loads are given by

$$p_c = \frac{EI}{R^3}(N^2-1), \tag{1}$$

where $N \geq 2$ represents the number of the axes of symmetry of the non-circular shapes.

Linear (elastic) materials are characterized by the relationship

$$\sigma = E\varepsilon \tag{2}$$

where σ, E and ε are the stress, Young's modulus and the strain, respectively. With this assumption an elastic tube deforms on applied stress and returns to its original shape as the stress is removed. Microscopically, this deformation involves stretching of the molecular network without slipping the atoms past each other and the molecular structure remains unchanged before and after stress, see [6].

A strain hardening or work hardening is a process to strengthen the material through the plastic deformation. Macroscopic plastic deformation is associated with dislocation within the microstructure of the material. The dislocation of material microstructure can result in additional strengthening of the material, therefore greater stress will be required to initiate plastic deformation (see, for example, [6,7]).

During work hardening of material mechanical deformation brings the material into plastic domain and material behaves nonlinearly. This behavior exhibits a nonlinear relationship between stress σ and strain ε. One common mathematical description of the work hardening phenomenon for an isotropic material is the

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http://dx.doi.org/10.1016/j.tws.2013.10.007

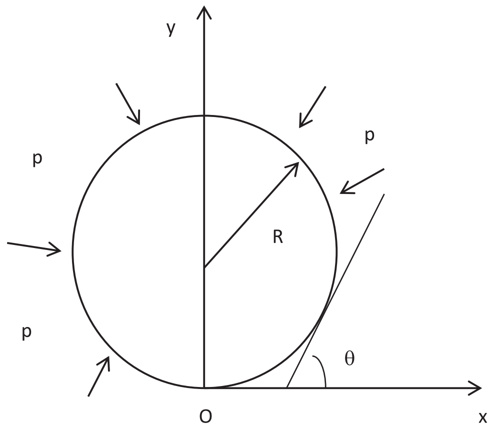

Fig. 1. A thin ring under uniform external pressure.

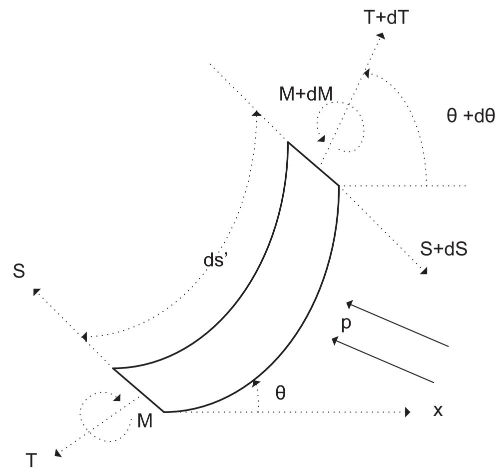

Fig. 2. Forces and moments acting on an infinitesimal element.

Hollomon's stress–strain equation:

$$\sigma = K|\varepsilon|^{n-1}\varepsilon, \quad 0 < n \le 1 \tag{3}$$

where K is the Bulk modulus, n is the strain hardening exponent. Materials which are described by the above power-law are sometimes called Hollomon or Ludwick materials. Values for K and n for many materials can be found in some engineering text books (see for example [6,8,9]). Buckling and deformation of nanotubes made of nonlinear material have been the subject of many recent articles (see for example [10] and the reference there in). In the rest of this paper, due to symmetry, we consider the deformation of a typical cross section of the tube, that is, the deformation of a circular, thin, inextensible ring subject to an external uniform pressure acting normally in the plane of the ring (see Fig. 1).

The layout of the paper is as follows. In Section 2 we present the equilibrium equations for a Hollomon's power-law material ring under uniform external pressure as a differential system of equations. In Section 3 we present a perturbation analysis valid for small deformation, and show that the buckling loads for a Hollomon's power-law material ring are given by

$$p_c(n) = \frac{nKI_n}{R^{2+n}}(N^2-1). \tag{4}$$

In Section 4 we present some numerical simulations for the post-buckling behavior of a Hollomon's power-law material ring for various values of the strain hardening exponent n. In Section 5 we give some concluding remarks.

2. Mathematical formulations

We consider the deformation of a thin, inextensible, circular ring made of high strength material that follows the Hollomon's law (3), under a uniform external pressure p acting normally on the ring, see Fig. 1.

For small $p > 0$, then the ring remains circular. As p exceeds a certain critical pressure p_c, the first buckling load, the circular solution becomes unstable and the ring deforms into buckled states.

As it is done in [1,3,9], balancing the forces and moments acting on an infinitesimal element ds (see Fig. 2), we obtain the (normalized) differential equations:

$$\frac{dx}{ds} = \cos\,\theta, \quad \frac{dy}{ds} = \sin\,\theta, \tag{5}$$

and

$$\frac{d^2M}{ds^2} + k\int k\frac{dM}{ds}ds - p = 0, \tag{6}$$

where $(x(s), y(s))$ are the coordinates of a point on the ring, M is the moment, $k = d\theta/ds$ is the curvature.

For linear material the moment is directly proportional to the curvature. For non-linear material, however, the relationship is more complex. To this end if z be the distance from the neutral axis then the strain ε is given by

$$\varepsilon = zk, \tag{7}$$

while the bending moment M is given by

$$M = \int_A \sigma z\, dA. \tag{8}$$

Using (7) and (3) in (8), we get

$$M = \int_A K|zk|^{n-1}zk \cdot z\, dA. \tag{9}$$

The nth moment of inertia I_n is described (see [5]) by

$$I_n = \int_A z^{n+1}\, dA,$$

which together with (8) give

$$M = KI_n|k|^{n-1}k. \tag{10}$$

Eq. (10) may be referred as the generalized Euler–Bernoulli moment of inertia for a high strength material ring. Similar equations have been derived in [11,12] in the study of buckling of column and beams. Note that for $n=1$ Eq. (10) reduces to the usual Euler–Bernoulli moment of inertia for a thin elastic ring.

Eqs. (5), (6) and (10) give the equilibrium equations:

$$\frac{d^2}{ds^2}(|k|^{n-1}k) + k\int k\frac{d}{ds}(|k|^{n-1}k)\,ds - p = 0, \quad 0 < n \le 1, \tag{11}$$

with normalization by R on arc length s and by KI_n on pressure p, and

$$\frac{dx}{ds} = \cos\,\theta, \quad \frac{dy}{ds} = \sin\,\theta. \tag{12}$$

We observe that if $k \ge 0$ then (11) reduce to

$$\frac{d^2}{ds^2}k^n + \frac{n}{n+1}k^{n+2} - ck - p = 0, \quad 0 < n \le 1, \tag{13}$$

where c is an arbitrary constant for integration. For $N \ge 2$ axes of symmetry the associated boundary conditions are

$$k'(0) = 0, \quad k'\left(\frac{2\pi}{N}\right) = 0, \quad \int_0^{2\pi/N} k(s)ds = \frac{2\pi}{N}. \tag{14}$$

The constant c is to be determined alone with the solutions of (11) and (12) subject to the conditions in (14) and $x(0) = y(0)$.