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Modal decomposition of thin-walled member collapse mechanisms

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ABSTRACT

Following recent investigations on the decomposition of elastic buckling modes into combinations of structurally meaningful deformation modes, this work presents a novel extension of the above procedure to elastic–plastic collapse mechanisms and highlights the relevant role that this concept may play in the mechanical knowledge/interpretation of thin-walled member failures. In order to achieve the sought decomposition, a code based on a Generalised Beam Theory (GBT) formulation developed to perform first-order elastic–plastic analyses of thin-walled members is employed. Five illustrative examples are presented and discussed, and the results displayed, namely load-deflection curves, deformed configurations and stress contours, are validated through the comparison with values provided by shell finite element analyses. The most relevant modal results addressed consist of (i) load-deflection curves determined on the basis of pre-selected deformation mode sets, (ii) modal participation diagrams and (iii) modal amplitude functions. These results make it easy to characterise and interpret the mechanics associated with the thin-walled member elastic–plastic failures (as well as with the various loading stages), which may be of great importance in the improvement/development of existing/new design methods (*e.g*, yield-line theory, direct strength method).

1. Introduction

It is well known that the structural behaviour of the vast majority of thin-walled steel members is genuinely non-linear, both physically and geometrically. Moreover, the ultimate strength and collapse mechanism of such members are invariably governed by a combination of plasticity and instability effects, which cause local and/or global deformation patterns. In a broad sense, the kinematical definition of local and global deformation patterns (and instability phenomena) is the following:

(i) Global deformation patterns are characterized by the fact that the member cross-sections experience in-plane rigid-body motions, namely transverse translations, associated with bending, and/or twist rotations, associated with torsion.[1] Therefore, the member deformation pattern can be fully defined by a combination of displacements/rotations along the member axis, combined with the so-called cross-section warping displacements.
(ii) Local deformation patterns differ from their global counterparts in the fact that cross-section walls exhibit in-plane deformations. It is still possible to distinguish between local-plate (wall transverse bending with no corner in-plane motions) and distortional (wall transverse bending combined with rigid-body motions of cross-section parts that involve corner in-plane motions) deformation patterns – when no confusion is bound to arise, the local-plate deformation patterns are merely termed as “local”.

Since thin-walled members usually have walls with small thickness (very slender), their ultimate strength behaviour is predominantly governed by instability (buckling) effects. Thus, it is just natural to establish a correspondence between the deformation patterns at failure with the local and global elastic buckling mode shapes, taking advantage of the very large amount of available research work dealing with the buckling and ultimate strength behaviour of thin-walled steel members, namely those made of cold-formed profiles [1–6]. In recent years, and since the elastic post-buckling stiffness (PBS) of a given member is highly dependent on its critical buckling mode nature (global – low PBS, local – high PBS, distortional – intermediate PBS), research work has been performed on the mechanical definition and classification of elastic buckling modes [7–10]. However, the first method that genuinely took into account (or, better, was based on) the modal nature of the thin-walled member behaviour was Generalised Beam Theory (GBT) (e.g., [11]). In GBT, the degrees of freedom are the nodal values and derivatives of amplitude functions providing the variation, along the member length, of pre-determined

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[1] Of course, the (global) axial extension deformation pattern is an exception to this rule.

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cross-section deformation modes with various natures (e.g., global, local, and distortional). A given deformed configuration is expressed as a linear combination of products involving each of the above deformation modes and its amplitude function. In the last few years, the Constrained Finite Strip Method (cFSM) was developed with the aim of incorporating the GBT modal characteristics into the classical finite strip analysis. Generally speaking, the cFSM imposes kinematic constraints to the displacement field of the standard semi-analytical finite strip method, thus obtaining the decomposition of a given buckling mode into a linear combination of deformation modes.

Unlike in buckling analysis, the modal decomposition of equilibrium/collapse configurations in the context of an inelastic member analysis has been very scarcely investigated up to now (some preliminary work on this topic was reported in [12]). A great advantage of this approach is to enable a fairly simple characterisation of the mechanics associated with the elastic–plastic failure of a thin-walled member, thus making it possible to improve/develop the existing/new design methods for such members – for instance, the application of the Direct Strength Method (DSM [13]) can benefit considerably from the output of this approach. So far, only shell finite element analyses (SFEA) have been capable of accurately assess the inelastic behaviour/strength of thin-walled members, due to the inherent complexity associated with the interaction between instability and/or plasticity effects. In order to overcome the SFEA drawbacks (high computational cost, error-prone data processing and complex result interpretation), considerable research work has been carried out on the development of yield-line mechanisms (based on rigid-plastic analysis) intended to approximate the true (elastic–plastic) collapse modes occurring in thin-walled members and, thus, provide the means to estimate the corresponding failure loadings [14–19] – Fig. 1, which shows the yield-line mechanisms of a plain channel member subjected to positive and negative minor-axis bending, illustrates this approach. However, due to its intrinsic nature (plasticity is assumed to occur exclusively along the yield-lines, while the remaining zones exhibit only rigid-body motions), these mechanisms can only provide an upper bound of the member ultimate strength. Moreover, significant challenges must still be overcome before this approach can be routinely applied in practical applications – for instance, the need for (i) an *a priori* definition of the spatial collapse mechanism, accounting for the influence of residual stresses and geometrical imperfections, and (ii) a correct definition of the yield-line bending strengths (this information must be available for different member geometries, material behaviours, loadings and support conditions).

The aim of this paper is to propose and illustrate the application of a GBT-based concept of modal decomposition of thin-walled member deformed configurations in the elastic–plastic range, including collapse mechanisms. In order to achieve this goal, a first-order elastic–plastic GBT formulation recently developed and numerically implemented by the authors [20] is adopted. After providing a very brief overview of this GBT formulation, its application to the modal decomposition of thin-walled member deformed configurations is illustrated by means of results concerning five different thin-walled members. Some of the GBT-based results obtained, namely load-deflection curves, deformed configurations and stress contours, are validated against values provided by ABAQUS [21] SFEA. The most relevant modal results addressed consist of (i) load-deflection curves, determined by means of GBT analyses that include only pre-selected deformation mode sets, (ii) modal participation diagrams and (iii) modal amplitude functions.

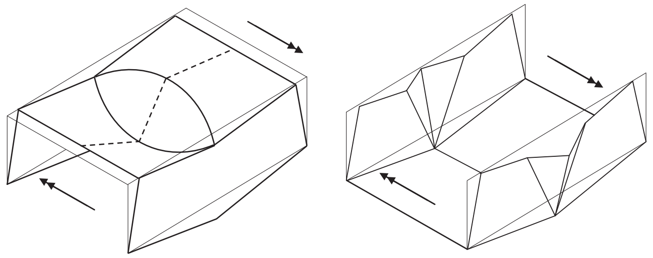

Fig. 1. Plain channel beam under positive and negative minor-axis bending: yield-line collapse mechanisms.

2. Brief overview of GBT

Consider the local coordinate system (x, s, z) defined at each thin-walled member wall mid-surface, as illustrated in Fig. 2, where x, s and z are, respectively, the longitudinal ($0 \le x \le L$ – L is the member length), mid-line transverse ($0 \le s \le b$ – b is the wall width) and through-thickness($-t/2 \le z \le t/2$ – t is the wall thickness) coordinates. The corresponding local displacements are (i) u (along x – warping), (ii) v (along s – transverse) and (iii) w (along z – flexural). The GBT analysis of a structural member consists of two main steps, namely (i) a cross-section analysis and (ii) a member analysis. The cross-section analysis performed in this study is the one developed by Silva et al. [22,23], which considers four deformation mode families: (i) conventional (global, local and distortional) modes, (ii) warping shear modes, (iii) transverse extension modes and (iv) cell shear flow modes – the latter come into play only in members with cross-sections exhibiting one or more closed cells. These deformation modes are obtained by solving a sequence of eigenvalue problems [22,23]. Fig. 3, which concerns a lipped angle cross-section, shows the in-plane and out-of-plane configurations of deformation modes belonging to three of the above four families (there is no closed cell). Each deformation mode is associated with a unique displacement profile involving in-plane ($v_k(s)$ and $w_k(s)$) and out-of-plane ($u_k(s)$ – warping) displacements, all functions of the mid-line coordinate s.

The member analysis comprises the determination of the modal amplitude functions $\zeta_k(x)$ that provide the variation of each deformation mode amplitude along the member axis (coordinate x). Fig. 2 shows a qualitative and schematic representation of three modal amplitude functions corresponding to the torsion distortional and local deformation modes.

In a GBT analysis, the displacement field at the member mid-surface is expressed as

$$u(s,x) = u_k(s)\zeta_{k,x}(x) \quad v(s,x) = v_k(s)\zeta_k(x) \quad w(s,x) = w_k(s)\zeta_k(x), \tag{1}$$

where (i) $u_k(s)$, $v_k(s)$ and $w_k(s)$ are the (normalised[2]) deformation mode displacement profiles, (ii) $\zeta_k(x)$ are the corresponding modal amplitude functions and (iii) the summation convention applies to subscript k. Thus, the displacement field associated with any member deformed configuration (e.g., a buckling mode or a collapse mechanism) is expressed as a linear combination of products involving modal displacement profiles and their longitudinal amplitude functions.

Only a brief overview of the GBT formulation to perform first-order elastic–plastic analyses is presented in this section – the interested reader may find detailed information about this formulation in Ref. [20]. The determination of a non-linear equilibrium configuration requires the use of an incremental–iterative strategy (the cylindrical arc-length method was used in this work)

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[2] The axial extension and warping shear deformation modes are normalised so that they exhibit a unit maximum axial displacement. All other deformation modes are normalized to exhibit a unit maximum in-plane displacement.