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A novel finite volume based formulation for the elasto-plastic analysis of plates

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ABSTRACT

In this paper, a novel finite volume formulation for the elasto-plastic analysis of Mindlin–Reissner plates is proposed. A layered approach is adopted which enables to monitor the evolution of the through-thickness plasticity. For the solution of the discretized equations, two different incremental solution algorithms are implemented. The proposed method is validated through a series of benchmark comparisons. It is observed that the results obtained are in good agreement with the reference results. It is also demonstrated that the proposed finite volume based formulation has good capabilities in dealing with the plastic modeling of very thin to moderately thick plates.

1. Introduction

In recent years, considerable efforts have been made in the development of numerical methods for the analysis of solid mechanics problems. The elasto-plastic behavior of plates has been analyzed by numerical methods such as the finite difference [1,2], discrete method [3], finite element [4–9], finite strip [10–12], boundary element [13,14], meshless based methods [15,16] and others.

Recently, there has been growing interest in developing finite volume (FV) discretization method for the solution of solid mechanics problems. Elastic analysis of three dimensional solids [17], stress analysis of elasto-plastic solids [18], bending analysis of elastic plates [19–21] are among them. FV formulation for the plate analysis has some advantages [20,21]:

- It is simple and transparent
- It behaves well in the analysis of very thin to thick plates
- It predicts accurate results without using any adjustable parameter for thin plates

Two types of finite volume technique are common: cell vertex finite volume [17,20] and cell centered finite volume [18–22].

In this paper, a finite volume based formulation is presented for the elasto-plastic bending analysis of plates in which the Mindlin–Reissner plate theory is used. Mindlin–Reissner plate theory accounts for the transverse shear effects and therefore, it can be applicable for both thin and thick plates. The present formulation is an extension of the previously developed finite volume formulation by the first author which deals with the elastic behavior of plates [20,21]. A cell centered type of finite volume is used in which the computational points are considered at the cell centers. In order to track the development of plasticity through the plate thickness, a layered approach is utilized. The layers are assumed to have isotropic and homogeneous material properties, to be perfectly joined and to have constant thickness. An incremental solution procedure is applied in which the status of each layer is evaluated by comparing the acting stresses with the yield criteria in each load step. The Von Mises yield criteria is used in which the effects of transverse shear stresses in plastic behavior is ignored. In order to cancel out the effects of the out of balance forces generated by bringing those stress points which moved out of the yield surface onto the yield surface, two iterative procedures are used. In the first approach, in order to satisfy the equilibrium state, the load increment is adjusted by finding a specific load factor. In the second approach the standard Newton–Raphson method [4] is applied for the above purposes.

For the evaluation of the proposed approach, a series of benchmark problems are studied. Findings of this work illustrate the capability of the finite volume method for the elasto-plastic analysis of very thin to thick plates.

This paper is presented in 6 sections. After Section 1, plate formulation based on the cell centered finite volume technique is presented in Section 2. Section 3 describes the solution of the discretized equilibrium equations. In Section 4, the layered approach is presented for the modeling of the plastic behavior of

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plates. Section 5 deals with the some benchmark tests that are analyzed using the present formulation. Finally, in Section 6, we draw the conclusions.

2. Plate formulation with cell centered finite volume method

We consider a plate with a mid-plane laid in xy plane and the outer surfaces of the plate are at $z = \pm t/2$ where t is the thickness of the plate. Displacements of a point of the plate along x, y and z axes are denoted by u, v and w respectively. Based on the Mindlin–Reissner plate theory, also known as the first order shear deformation theory (FSDT), the above displacement components of a plate's point can be expressed in terms of three independent variables as

$$u = z\beta_x(x,y)$$
$$v = z\beta_y(x,y)$$
$$w = w(x,y) \tag{1}$$

where z is the distance of the point from the mid-plane, β_x and β_y are rotations of the plate sections in the xz and yz planes respectively. The sign convention related to these rotations is presented in Fig. 1.

The stress resultant forces acting on an infinitesimal portion of the plate are also shown in Fig. 1 in which Q_x and Q_y are the transverse shear forces, M_x and M_y are the bending moments and M_{xy} is the twisting moment. All these resultants are measured per unit length of the plate.

In the cell centered finite volume method, the plate mid-plane is divided into the elements known as cells or control volumes [20,21]. The center of each cell is considered as the computational point where the unknown variables are allocated. The equilibrium conditions of each cell are expressed in terms of the stress resultants acting on cell faces. In presenting the equilibrium equations, it is assumed a continuous variation of moments and shear forces over cell faces. An assumption that can be implemented and simplifies the discretization procedures is assuming a constant moment and shear forces within the region surrounded by dashed lines shown in Fig. 2a.

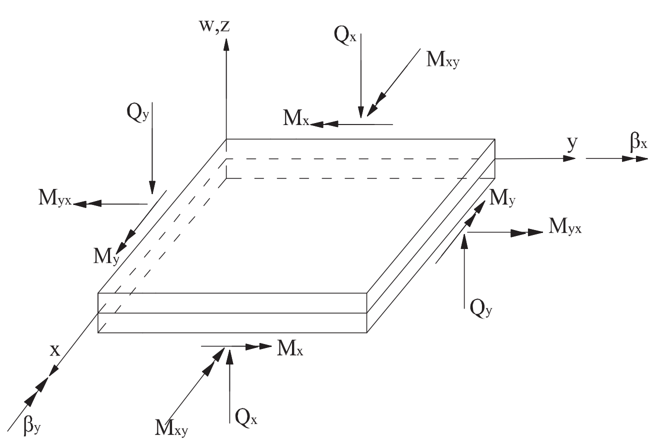

Fig. 1. sign convention of moment, shear force and section rotation.

The equilibrium state of a typical control volume lying in the xy plane can be expressed as

$$\sum \begin{bmatrix} M_x \\ M_y \\ F_z \end{bmatrix} = 0 \tag{2}$$

where the first two equations express the equilibrium of moments about the x and y axes, respectively, and the third equation expresses the equilibrium state in the z direction. Based on the aforementioned assumptions relevant to the moments and shear forces distributions, and using the sign convention presented in Fig. 1, the equilibrium Eq. (2) can be re-expressed as follows:

$$\sum_i^m \left\{ \begin{bmatrix} 0 & n_y^i & n_x^i \\ n_x^i & 0 & n_y^i \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x^i \\ M_y^i \\ M_{xy}^i \end{bmatrix} - \begin{bmatrix} n_x^i(Y_i) & n_y^i(Y_i) \\ n_x^i(X_i) & n_y^i(X_i) \\ n_x^i & n_y^i \end{bmatrix} \begin{bmatrix} Q_x^i \\ Q_y^i \end{bmatrix} \right\} L_i = \begin{bmatrix} 0 \\ 0 \\ qA_p \end{bmatrix} \tag{3}$$

where i varies from one to number of faces of the cell, n_x^i and n_y^i are cosine directions of outward normal of face i, $X_i = (x_i - x_p)$ and $Y_i = (y_i - y_p)$ where x_i and y_i are the coordinates of the midpoint of face i, x_p and y_p are the coordinates of cell center (computational point), q is the uniformly distributed load applied upon the cell with the mid-surface area of A_p, L_i is the length of face i, M^i and Q^i are moment and shear force corresponding to face i, respectively that are measured per unit length. The constitutive equations are given in the form of

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \mathbf{D}^* \begin{Bmatrix} \frac{\partial \beta_x}{\partial x} \\ \frac{\partial \beta_y}{\partial y} \\ \frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x} \end{Bmatrix}, \quad \mathbf{D}^* = D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{4a, b}$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = kGt \begin{Bmatrix} \frac{\partial w}{\partial x} + \beta_x \\ \frac{\partial w}{\partial y} + \beta_y \end{Bmatrix} \tag{5}$$

where D is the plate flexibility rigidity, G is the shear modulus, ν is Poisson's ratio and k is lateral shear correction factor which is due to assumption of constant transverse shear strains.

To calculate the derivations of displacement components in global coordinate system, xy, a local coordinate system, $\xi\eta$, can be used where lines Pi and N_1N_2 are designated as local co-ordinate axes ξ and η, respectively, see Fig. 2. Using the chain rule

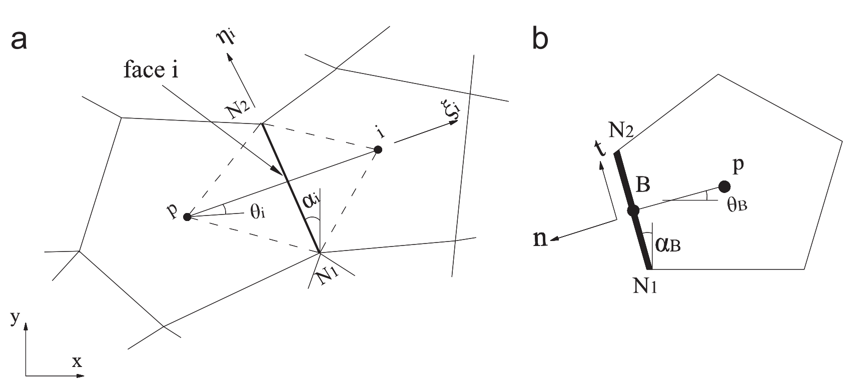

Fig. 2. (a) cell center control volume and (b) control volume adjacent to the boundary.