Thin-Walled Structures 77 (2014) 174–186

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Accurate dynamic response of laminated composites and sandwich shells using higher order zigzag theory

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ARTICLE INFO

Article history:
Received 18 March 2013
Received in revised form
20 August 2013
Accepted 3 September 2013
Available online 30 October 2013

Keywords:
Composite
Sandwich
Shell
Forced vibration
Higher order zigzag theory
Boundary condition

ABSTRACT

Forced vibration response of laminated composite and sandwich shell is studied by using a 2D FE (finite element) model based on higher order zigzag theory (HOZT). This is the first finite element implementation of the HOZT to solve the forced vibration problem of shells incorporating all three radii of curvatures including the effect of cross curvature in the formulation using Sanders' approximations. The proposed finite element model satisfies the inter-laminar shear stress continuity at each layer interface in addition to higher order theory features, hence most suitable to model sandwich shells along with composite shells. The C_0 finite element formulation has been done to overcome the problem of C_1 continuity associated with the HOZT. The present model can also analyze shells with cross curvature like hypar shells besides normal curvature shells like cylindrical, spherical shells etc. The numerical studies show that the present 2D FE model is more accurate than existing FE models based on first and higher order theories for predicting results close to those obtained by 3D elasticity solutions for laminated composite and sandwich shallow shells. Many new results are presented by varying different parameters which should be useful for future research.

1. Introduction

Laminated composite and sandwich shell structures are widely used in civil, mechanical, aerospace and other engineering applications. Laminated composite and sandwich materials are becoming popular because of their high strength to weight and strength to stiffness ratio. The most important feature for the analysis of laminated composite and sandwich structure is that the material is weak in shear compared to extensional rigidity. Due to this reason transverse shear deformation of the composite and sandwich shell has to be modeled very accurately.

In the earlier days, Love [1] started the small amplitude free vibration study of thin elastic shells and postulated [2] the first approximation of the classical thin shell theory. Many studies related to vibration of shells have then been carried out. An excellent summary of works on the vibration aspects of thin shells before the 1970s was presented in a monograph by Leissa [3]. It included a detailed account of the development and derivation of the various classical thin shell theories. Using Love's first approximation with different strain–displacement relations, various formulations describing the equations of motion of a vibrating thin circular cylindrical shell were discussed. Most parts of the monograph were devoted to the analysis of closed circular cylindrical shells having various boundary conditions, cut-outs, effects of added mass, anisotropy, variable thickness, initial stress, and other complicating factors.

Ganapathi et al. [4] used a higher order theory to perform dynamic analysis of laminated cross-ply composite non-circular thick cylindrical shells. Khare and Rode [5] utilized a higher order theory to develop closed-form solutions for vibrations of thick shells. Pradyumna and Bandyopadhyay [6] studied the static and free vibration behavior of laminated composite shells based on a higher-order shear deformation theory (HSDT). Liew and his colleagues [7–10], Kumar et al. [11] carried out extensive research work on the vibrations of different types of shell surfaces including spherical, cylindrical, hyperbolic paraboloidal, and trapezoidal shells using the Ritz's minimization technique to obtain the natural frequency and mode shapes.

Liew and Lim [12,13] proposed a higher-order theory by considering the Lame parameter $(1+z/R_x)$ and $(1+z/R_y)$ for the transverse strains, which were neglected by Reddy and Liu [14]. This theory accounts for cubic distribution (non-even terms) of the transverse shear strains through the shell thickness in contrast with the parabolic shear distribution (even-terms) of Reddy and Liu [14]. Liew et al. [15] documented the developments in the free vibration analysis of thin, moderately thick, and thick shallow shells. Kumar et al. [16,17], Reddy and Liu [18], and Kant and Menon [19] also presented higher-order theories for composite and sandwich cylindrical shells.

Other thick shell theories, such as layer-wise theories have also been utilized. These theories typically reduce a 3D problem to a 2D

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http://dx.doi.org/10.1016/j.tws.2013.09.026

problem by expanding the 3D displacement field in terms of a 2D displacement field and the through-the-shell thickness. Basar and Omurtag [20] used a layer-wise model to investigate free vibrations of shell structures. Other research involving layer-wise shell theories include Moreira et al. [21] who used a layer-wise theory to formulate shell finite elements for dynamic modeling of composite laminates.

The typical feature of sandwich panel is that the variation of in-plane displacements across the thickness shows kink at the interface between the core and face layer, which gives discontinuous transverse shear strain at these interfaces. This phenomenon is also found in laminated composite shells for its layered construction but the discontinuity is not so prominent as that of sandwich shell. Actually, it depends on the difference in the values of transverse shear rigidity and thickness of adjacent layers, which is quite significant in sandwich shell compared to that of composite laminates. However, the effect of this discontinuity cannot be ignored in a multilayered thick composite laminate. It is observed that increase in the order of polynomial across the thickness (i.e. HSDT), for the in-plane displacements cannot make any significant improvement in predicting transverse shear stresses accurately and hence unable to capture real behavior of multilayered laminates. Actually, it gives a continuous variation of shear strain across the thickness, which gives discontinuity in the shear stress distribution at the layer interfaces due to different values of shear rigidity at the adjacent layers. But the actual phenomenon is just opposite i.e. the shear strain is discontinuous and shear stress is continuous at the layer interfaces. In order to consider this aspect, the layer-wise theories are proposed. In these theories the unknowns are taken at all the layer interfaces, which give a zigzag pattern of through thickness variation for the in-plane displacement to represent the desired shear strain discontinuity at the layer interfaces. In terms of solution accuracy, the performance of these theories is very good but they require a huge computational effort as the number of unknowns increase with the increase in the number of layers.

In some earlier studies sandwich structures are characterized by a soft core between two stiffer faces. In the framework of mixed multilayered plate theories, Murakami [22] proposed a Zig-Zag Function (ZZF) able to reproduce the described slope discontinuity. Equivalent single layer models (the layers are treated as one-layer equivalent plate) with only displacement unknowns can be developed on the basis of ZZF. The advantages of using the ZZF to analyse the multilayered anisotropic plates and shells have also been discussed by Carrera [23]. These discontinuities make difficult the use of classical theories such as Love [2] type theories. In order to trace an accurate vibration response of sandwich structures layer-wise models where three independent layers are considered and used to capture the above ZZ form. In this direction, the works of Burton and Noor [24], Noor, Burton and Bert [25], Altenbach [26], Librescu and Hause [27] and Vinson [28] may be referred. These models are computationally expensive and cannot model the modern laminated face sheets having many layers.

Carrera [29] compared more than 50 available theories and finite elements to those developed in the framework of the unified notation. Carrera [29] addressed closed form solutions and finite element results, zigzag effects and inter-laminar continuity in laminated plates and shells. Carrera and Ciuffreda [30] used a unified formulation to compare about 40 theories for multilayered, composites and sandwich plates which are loaded by transverse pressure with various in-plane distributions. Cinefra et al. [31] used nine nodes shell finite element employing refined models based on the CUF (Carrera Unified formulation) for the static analysis of plates and shells made of functionally graded material. Demasi [32] presented partially zigzag advanced higher order shear deformation theories based on the GUF (Generalized Unified Formulation). However, CUF, GUF and higher order theory by Kant and Menon [19] contained some parameters which have no physical significance; hence it is difficult to incorporate the boundary conditions.

The analysis of the forced vibration of composite structures has been studied by many researchers in recent times. Mainly analytical solutions are proposed by investigators for the case of simply supported edge conditions using different procedures. Sun and Whitney [33–35], Dobyns [36], and Khdeir and Reddy [37–38] used the classical separation of variables with the Mindlin–Goodman procedure whereas Chou [39] used the Laplace transform technique. The Newmark direct integration method was used by Bhimaraddi [40–41]. Khdeir et al. [42] developed an exact approach using the state space technique. These results obtained by using closed-form solutions for the dynamic response of laminated plates and shells were for simply supported boundary condition. Therefore, a general solution technique like FE method is needed to obtain solutions for other boundary conditions.

It has been observed that the in-plane displacements actually show a piecewise parabolic variation across the thickness of the shell to retain the possibility of strain jumps at the layer interfaces for dissimilar materials in two adjacent layers. To model this zigzag pattern of in-plane displacement fields for the laminated composite structures, 2D higher order zigzag theories (HOZT) are developed which gives results very close those obtained by using a 3D model. Therefore, HOZT should be one of the best 2D models for the forced vibration analysis of laminated composite structures for the accurate prediction of forced response.

It is observed that there is no literature available on forced vibration of composite and sandwich shells based on higher order zigzag theory incorporating all three radii of curvatures by using a suitable finite element model.

Considering all the above aspects, a C_0 2D finite element shell model based on higher order zigzag theory incorporating all three radii of curvatures which has been developed by authors [43–44] recently for static and free vibration respectively, is extended to the study of the forced vibration analysis of composite and sandwich shell panels.

It gives parabolic through thickness variation of transverse shear strains with discontinuity at the layer interfaces as desired. It also gives zero shear strains at the top and bottom surfaces of the laminate. Thus it possesses all the features required for accurate modeling of composites and sandwich laminates. The problem of C_1 continuity associated with the theory has also been overcome in this model and a C_0 isoparametric finite element has been formulated for this purpose. The element contains nine nodes with seven unknowns at each node. The analysis has been performed considering shallow shell assumptions. The effect of all the three radii of curvature is also included in the formulation. The present finite element model based on higher order zigzag theory (HOZT) is applied to solve many problems on free vibration of composite and sandwich shells considering different shell geometries and boundary conditions, etc. The present results are also validated with some published 3D elasticity results and higher order theory results.

2. Theory and formulation

The in-plane displacement fields (Fig. 1) are taken as below:

$$u_\alpha = u_\alpha^0 + \sum_{k=0}^{nu-1} S_\alpha^k (Z - Z_k^u) H(Z - Z_k^u) + \sum_{k=0}^{nu-1} T_\alpha^k (Z - Z_k^l) H(-Z + Z_k^l) + \xi_\alpha Z^2 + \phi_\alpha Z^3 \quad (1)$$

where u_α^0 denotes the in-plane displacement of any point on mid surface, nu and nl represent number of upper and lower layers