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A theoretical analysis of the local buckling in thin-walled bars with open cross-section subjected to warping torsion

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ABSTRACT

Results of a theoretical analysis of the local buckling in thin-walled bars with open cross-section subjected to warping torsion are presented. The local critical bimoment, which generates local buckling of a thin-walled bar and constitutes the limit of the applicability of the classical Vlasov theory, is defined. A method of determining local critical bimoment on the basis of critical warping stress is developed. It is shown that there are two different local critical bimoments with regard to absolute value for bars with an unsymmetrical cross-section depending on the sense of torsion load (sign of bimoment). However, for bars with bisymmetrical and monosymmetrical sections, the determined absolute values of local critical bimoments are equal to each other, irrespective of the sense of torsional load. Critical warping stresses, local critical bimoments and local buckling modes for selected cases of thin-walled bars with open cross-section are determined.

1. Introduction

Cold-formed thin-walled bars with open cross-section belong to groups of members in which limit load-carrying capacity is predetermined by local or distorsional buckling. Bending and nonuniform torsion occur in thin-walled steel beams in which transverse load acts off the shear center of the cross-section. Torsional moments and bimoments appear in cross-sections in the process of lateral-torsional buckling or flexural-torsional buckling of thin-walled members with geometrical (general and local) imperfections. In this case, torsion moments and bimoments are generated by an amplification of displacements and angles of rotation “along the directions” of geometrical imperfection. Contemporary cold-formed steel members, or beams welded from thin sheet metals with open section, are characterized by small thickness of walls and relatively small torsional rigidity. Bimoment caused by warping torsion can be an essential component of section load for this class of thin-walled members.

For the purpose of precise description of phenomena occurring in thin-walled bars with open cross-section subjected to warping torsion, the following definitions will be applied in the further part of the study:

- A thin-walled bar “with rigid cross-section contour” – a thin-walled bar whose cross-sections during load increment are subject to warping displacement, but maintain the original shape of section contour;
- A thin-walled bar “with flexible cross-section contour” – a bar built from flat walls (thin plates) in which, after local or distortional critical stresses are reached, local deflections of component plates or displacements of stiffened edges of walls occur. As a result, the geometry of the cross-section contour of a thin-walled bar is changed;
- “A thin-walled bar segment” – the section of a bar between transverse stiffenings (diaphragms, ribs, etc.), which assure a rigid cross-section contour in place of their location.
- “The constructional system of a thin-walled bar” – the mutual geometrical arrangement of component plates (walls), transverse stiffenings (diaphragms, ribs), and local and overall boundary conditions of a thin-walled bar.

The Vlasov theory [1] refers to thin-walled bars with a rigid cross-section contour. This fact limits the possibility of its application to the estimation of the limit load-carrying capacity of currently used thin-walled bars with a flexible cross-section contour in which phenomena caused by local or distorsional buckling of walls occur. It is not possible to analyze post-buckling load-carrying capacity reserves from the Vlasov theory because local or distorsional buckling generates change in the geometry of the cross-section contour. Invariability of the cross-section contour is the fundamental assumption of the Vlasov theory [1].

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Nomenclature

A_{inp} coefficients of power polynomials
b_s, t_s width, thickness of a plate (wall s)
$B(x)$ bimoment function
$B_{cr,L}$, $B_{cr,R}$ local critical bimoment ("left" – positive, "right" – negative)
B_y first yield bimoment
D_s plate flexural rigidity (wall s)
E Young's modulus of elasticity
f_{ins}, f_{jqs} dimensionless, free parameters of deflection function of a plate (wall s)
G shear modulus of elasticity
i, j, n, q, p natural number subscript
i_o the number of half-waves of the sine function in the direction of the plate (or the segment) length
I_ω warping section constant
I_t St-Venant torsion constant
k_ω coefficient of critical warping stress
l_s length of a thin-walled bar segment, length of a plate (wall s)
L_s work done by external forces
L_{injq} component elements of the work done by external forces function
m coefficient which characterizes the longitudinal stress variation according to (19)
$M_{t,L}$, $M_{t,R}$ load of a concentrated torsional moment ("left" – positive, "right" – negative)
$M_{t,cr}$ critical torsional moment from the condition of local buckling
$M_1(x_s)$, $M_2(x_s)$ moments of elastically restrained longitudinal edges (No. 1, 2) of the component plate (wall s) in a thin-walled bar segment
n_o degree of the polynomial, number of polynomials
U sum of the total potential energy
V_s strain energy of the bending of a plate (wall s)
V_{injq} component elements of the bending strain energy function
w_s deflection function of a plate (wall s)
x_s, y_s, z_s Cartesian coordinates of a plate (wall s)
$\bar{x}$, $\bar{y}$, $\bar{z}$ Cartesian coordinates of a thin-walled bar segment
X_s longitudinal body forces
Y_{in} power polynomials (9) with previously determined coefficients A_{inp}
α_s coefficient of stress distribution in the direction of the width of a plate (wall s)
$\beta(x_s)$, $(\beta(\bar{x}))$ function of normal stress (or bimoment) distribution in the direction of the plate (or the segment) length
$\kappa = \sqrt{GI_s/EI_\omega}$ flexural – torsional coefficient of a cross-section
χ_s parameter describing load of a plate (wall s) on the edge containing the center of local coordinate system ($y_s = 0$)
$\varphi_1(x_s), \varphi_2(x_s)$ angles of a component plate's rotation (wall s) on longidudinal edges (No. 1, 2) at the connection of adjacent plates
ϕ angle of twist rotation
ν Poisson's ratio
ρ_s, δ_s geometrical parameters of a cross-section according to (4)
$\lambda_{ps} = b_s/t_s$ slenderness of a plate (wall s)
σ_c, σ_t normal stresses (compression, tension)
$\sigma_\omega, \tau_\omega$ warping stresses (normal, shear)
$\sigma^L_{cr}, \sigma^D_{cr}$ local buckling stress, distortional buckling stress
$\sigma^L_{\omega,cr}$ critical warping stress from the condition of local buckling (positive)
ω_i sectorial coordinates
ω_c a sectorial coordinate corresponding to critical stress $\sigma^L_{\omega,cr}$
$\sigma_{E,s}$ Euler's stress for a plate (wall s)
σ_0 comparative edge stress in the cross-section of a thin-walled bar
∇^2 Laplace's operator
Λ Lagrange's function for a thin-walled bar segment
ψ_q Lagrange's multipliers
μ_i multipliers of edge stresses

In practice, we can distinguish three types of sections of thin-walled bars built from flat walls whose behavior under load generating normal stresses is shown schematically in Fig. 1. Bars with a rigid cross-section contour, in which dependences: $\sigma_c < \sigma^L_{cr}$ and $\sigma_c < \sigma^D_{cr}$ (Fig. 1b) occur, i.e. bars with a flexible cross-section contour from the condition of local buckling: $\sigma^L_{cr} < \sigma_c < \sigma^D_{cr}$ or $\sigma^L_{cr} < \sigma^D_{cr} < \sigma_c$ (Fig. 1c), and bars with a flexible cross-section contour from the condition of distortional buckling for which $\sigma^D_{cr} < \sigma_c < \sigma^L_{cr}$ or $\sigma^D_{cr} < \sigma^L_{cr} < \sigma_c$ (Fig. 1d).

The necessity of the distinction of thin-walled bars with a rigid cross-section contour (Fig. 1b) from thin-walled bars with a flexible cross-section contour (Fig. 1c,d) under a load exceeding local or distortional critical stresses has an essential significance for the correct interpretation of phenomena

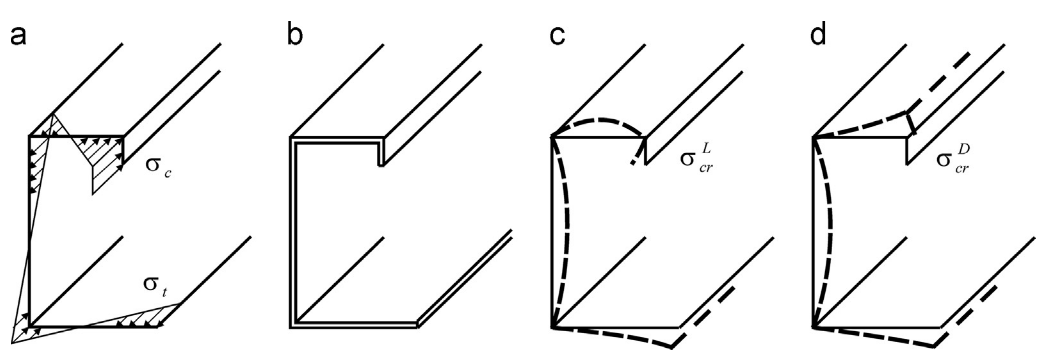

Fig. 1. Types of sections of thin-walled bars with flat walls (a) distribution of warping normal stresses, (b) rigid cross-section contour, (c) the flexible cross-section contour from the local buckling condition, and (d) the flexible cross-section contour from the distortional buckling condition.