Thin-Walled Structures 76 (2014) 65–76

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Problems in geometrically exact modeling of highly flexible beams

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ARTICLE INFO

Article history:
Received 6 May 2013
Received in revised form
31 October 2013
Accepted 17 November 2013
Available online 7 December 2013

Keywords:
Geometrically exact modeling
Highly flexible beams
Large rotations
Jaumann strains
Shear locking
Singularity of rotational variables

ABSTRACT

Because the deformed beam geometry often is the most important information for applications of highly flexible beams, a geometrically exact beam theory needs to be displacement-based in order to directly and exactly describe any greatly deformed geometry. Main challenges in geometrically exact beam modeling are how to describe a beam's large reference-line deformation and cross-sectional rotations without singularity and how to derive objective directional strains in terms of global displacements and rotations that contain elastic deformation and rigid-body movement. By comparing with a geometrically exact displacement-based beam theory this paper shows that theoretical and numerical problems of other geometrically nonlinear beam theories are mainly caused by: (1) use of independent variables to account for bending-shear rotations, (2) use of problematic energy-based Green–Lagrange strains in order to have objective strain measures, and/or (3) use of strain-based formulations in order not to use problematic Green–Lagrange strains. The theoretical problems include inconsistent governing equations from energy- and momentum-based formulations, inexistence of material property matrices for the chosen strain and stress measures, and non-directional stresses. The numerical problems include shear locking in finite-element analysis, the need of internal nodes and hence more degrees of freedom in finite-element modeling, singularity of mathematics-based rotational variables, deformed geometry being obtained by approximate post-processing numerical integration, and difficult to match secondary (force) variables with deformed conditions.

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1. Introduction

A *geometrically exact beam theory* can exactly describe a beam's arbitrarily large deformation and is needed for many engineering applications [1–8]. Many highly flexible beam-like medical devices and implants are used for today's surgical and medical treatments of different diseases [2,3]. Even in molecular biology, nonlinear beam theories are needed for modeling of supercoiling of DNAs in order to understand how it affects physical, chemical and biological properties of a molecule [4]. In petroleum engineering, a typical drillstring has a length around 5000 m and a length to diameter ratio around 10^5 [5], which is a slender beam even thinner than a human hair. Moreover, movie industry is developing toward natural looking dynamic animations using physics-based modeling and analysis, and beam dynamics plays a major role in such applications [6–8]. In order to advance theoretical structural mechanics for today's engineering and science applications, it is important to have a high-fidelity geometrically exact beam theory that can be used to investigate influences of high-order geometric nonlinearities on static and dynamic characteristics of highly flexible beam-like structures. However, challenges exist and most of the geometrically nonlinear beam theories in the literature are incapable of exact description of large beam deformations without theoretical and/or numerical problems [9–26]. Moreover, because the deformed geometry often is the most important information for engineering applications of flexible beams, a geometrically exact beam theory should be presented in terms of displacements, instead of stresses or strains [1,26].

Fig. 1 shows that a beam theory is to describe the deformed reference line and the deformed cross-section of a beam. Hence the three major tasks in modeling highly flexible beams are: (1) how to describe the reference-line deformation, (2) how to describe the cross-section deformation, and (3) how to derive directional objective strains in terms of global displacements and rotation variables. The reference-line deformation is independent of the cross-section deformation and can be fully described by the global, absolute displacements of points on the reference line [1]. The cross section is crooked mainly by out-of-plane shear warping. To better describe the cross-section deformation Timoshenko's beam theory improves the Euler–Bernoulli beam theory by accounting for assumed first-order transverse shear deformation, and third- and other higher-order beam theories improve Timoshenko's beam theory by accounting for the non-uniform distributions of transverse shear strains over the cross-section [1]. Moreover, layerwise higher-order

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http://dx.doi.org/10.1016/j.tws.2013.11.008

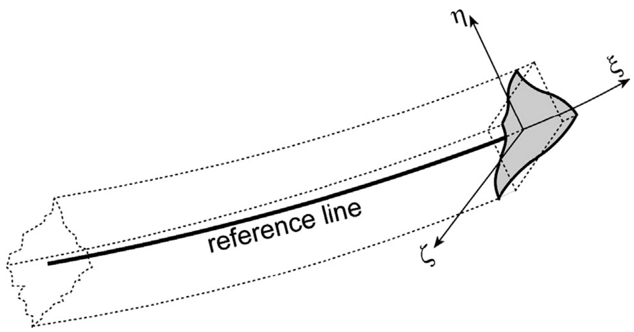

Fig. 1. A beam model consisting of a reference line and a cross section.

shear theories account for the non-uniform distributions of transverse shear strains through the layers of a composite laminate and the continuity of transverse shear stresses at the interface of any two adjacent layers, and quasi-3D beam theories improve layerwise higher-order shear theories by accounting for the transverse normal stresses caused by Poisson's effect and transverse normal loads [1]. Shear rotations are independent of bending rotations, and they may result in different acoustic and optical vibration modes [27]. Timoshenko's beam theory [28] is an order-deficient beam theory because it combines both bending and shear rotations into one bending-shear rotation angle [29]. This order deficiency results in the shear locking problem in finite-element analysis of thin beams, as shown later in Section 3.2.

For composite and built-up beams, some non-classical effects may become significant due to material anisotropy, asymmetry of the cross section, and/or different Poisson's ratios over the cross section. These effects include transverse shear deformation, torsional warping, inplane warpings due to bending and extension, transverse normal stresses, in-plane shear stresses, warping restraints at two ends, and the free-edge effect, and they cause significant in-plane and/or out-of-plane warping displacements and a 3D stress state. Hence, 3D finite element modeling may be the only way to solve dynamic problems of general composite beams, which is too expensive in order to achieve certain accuracy. Because in-plane and out-of-plane warpings are relative displacements with respect to the deformed cross section and are much smaller than global displacements, inertial forces caused by warpings are negligible. However, because these warpings offer extra degrees of freedom for the cross section to deform, they significantly affect the global stiffnesses of a beam. A combination of St. Venant's warping solutions derived from linear elasticity, and a 1D nonlinear beam model is natural and can account for 3D stress effects. More specifically, Berdichevskii [10] stated that the geometrically nonlinear problem of elastic beams can be decoupled into a nonlinear 1D problem and a linear 2D sectional problem. Consequently, one can neglect inertia forces due to in-plane and even out-of-plane warpings and only consider warpings in the calculation of elastic constants of beams. In other words, a 1D nonlinear beam model with global stiffnesses determined from a linear, static, 2D sectional analysis of warpings is a general and practical approach in solving nonlinear anisotropic beam problems [13,23]. Significant contributions toward accurate estimation of cross-sectional warpings and stiffnesses by 2D finite-element analysis of cross sections of isotropic and anisotropic beams were presented in [30–32]. Most of these geometrically nonlinear beam theories with cross-sectional stiffnesses obtained from 2D sectional analysis use bending-shear rotation angles and treat them as independent of global displacements, and hence they are prone to shear locking and singularity problems, as shown later in Sections 3.2 and 3.4.

A geometrically exact displacement-based beam theory enables nonlinear inverse design analysis of flexible beams, as shown later in Section 3.5. Moreover, a geometrically exact displacement-based beam theory is also valuable for investigating the boundary between linear and nonlinear response regions of a beam undergoing small- and moderate-amplitude vibrations. For example, recent research on system identification using advanced time–frequency analysis indicates that geometric nonlinearity can affect structural vibration characteristics even if the structure undergoes small-amplitude vibration with certain types of boundary/loading conditions [33]. For a regular on-earth 1D or 2D structure, its dynamic response is often pre-assumed to be linear if the vibration amplitude is less than its thickness. In perturbation analysis of geometrically nonlinear structures, governing differential equations are expanded into polynomials in terms of displacement variables and then the highest power of displacement variables is defined as the order of nonlinearity [34]. This also implies the use of vibration amplitude to define the magnitude of nonlinearity. However, it is more appropriate to define linear response to a harmonic excitation by giving a limit on the response's *continuous frequency bandwidth* instead of the vibration amplitude because nonlinear response can be due to a large vibration amplitude, boundary constraints, loading conditions, and other factors [33]. Different combinations of these factors may result in different nonlinear responses even if the displacement is smaller than the structure's thickness. For example, a hinged-hinged beam behaves more nonlinearly than a clamped-free beam under the same magnitude of vibration amplitude [33]. Moreover, a high-frequency mode starts to behave nonlinearly at an amplitude smaller than that of a low-frequency mode due to high curvatures. On the other hand, finite-element analysis using exact nonlinear strain–displacement relations shows that a cantilevered thin isotropic plate subject to a transverse corner load can behave linearly even when its transverse displacement is more than 10 times its thickness, but it behaves nonlinearly if von Karman strains (i.e., truncated nonlinear strain–displacement relations) are used (see Fig. 6.42 of [1]). Hence, geometrically exact high-fidelity modeling without Taylor expansion is really needed in order to use such structural theories for accurate static/dynamic analysis and for development of advanced techniques for inverse design, system identification, damage inspection, and health monitoring of structures.

In this paper, we first summarize an advanced total-Lagrangian geometrically exact displacement-based beam theory [35] that can exactly describe the reference-line deformation and cross-sectional rotations under any magnitude of displacements and rotations without singularity and has exact, explicit strain–displacement relations. Then this nonlinear beam theory is used to compare with and reveal, by in-depth derivations and reasoning, theoretical and numerical problems of other nonlinear beam theories that intend to be geometrically exact in the literature.

2. Geometrically exact displacement-based beam theory

To reveal problems of different geometrically nonlinear beam theories in the literature we summarize here a truly geometrically exact displacement-based beam theory [35]. An initially curved beam undergoing large deformation requires three coordinate systems to describe its movement, as shown in Fig. 2a. The *abc* is a fixed rectangular coordinate system used for reference, the *xyz* is a orthogonal curvilinear coordinate system used to describe the undeformed beam geometry, and the $\xi\eta\zeta$ is a moving orthogonal curvilinear coordinate system attached to the deformed beam. Let $\mathbf{i}_a$, $\mathbf{i}_b$ and $\mathbf{i}_c$ be the unit vectors of the *abc* system; $\mathbf{i}_x$, $\mathbf{i}_y$ and $\mathbf{i}_z$, and be the unit vectors of the *xyz* system; and $\mathbf{i}_1$, $\mathbf{i}_2$ and $\mathbf{i}_3$, be the unit vectors of the $\xi\eta\zeta$ system. Moreover, u, v and w represent the global, absolute displacements of the observed reference point O

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