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Optimal folding of cold formed steel cross sections under compression

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ABSTRACT

This paper aims at finding the optimal folding of open cold formed steel cross sections under compression. Starting with a fixed coil width, a design point in the design space is defined by a vector of turn angles at a set of points along the coil width. Generalized Finite Strip Method (FSM) and Direct Strength Method (DSM) are combined to calculate the nominal compressive strength for a given cross section (a given design with a given set of turn angles). The design space is searched primarily via a stochastic search algorithm, Genetic Algorithm (GA). The near-optimal folding of the cross section is then fine-tuned through a few steps of the gradient descent optimization. To arrive at practical designs the optimization problem is augmented with constraints on the geometrical properties of the cross section. The optimal cross sections are found to have compressive capacities that are higher than the original designs by a factor of more than three in many cases. The shape of the optimal folding is shown to be greatly influenced by the choice of boundary condition. Strategies for identification of instability modes, a necessary first step to using DSM, are also discussed in detail.

1. Introduction

Thin-walled structures are very common in engineering structures. Their application ranges from tiny machine parts to large structures such as fuselages, storage vessels and cooling towers. Among thin-walled structures, those that are composed of cold-formed steel (CFS) members have become widespread in the practice of structural engineering due mostly to the increasing demand for economical and efficient structural elements. Recent theoretical developments [1–3] along with the introduction of devoted open source software (e.g. CUFSM [4]) for the design and analysis of such CFS members have also lead to a more extensive usage of cold-formed steel in building industry, especially in the design and construction of low rise buildings.

Cold formed steel has many advantages over other construction materials. CFS members are lightweight. They weigh up to 35–50% less than their wood counterparts. High strength and stiffness to weight ratio is another advantage. This makes CFS members economical and the same time very easy to erect and install (almost no framework is needed). CFS is very durable, is not combustible, is easy to transport and handle, and can be easily recycled [5]. One of the most desirable features of CFS members, however, is that they may be shaped (cold-bent) to nearly any open cross section. This allows for the use of formal optimization strategies to find optimal shapes for the members' cross sections.

A number of scholars have conducted research on optimal cold-formed steel cross section selection and design. Lu [6] embedded CUFSM in a genetic algorithm routine to optimize Z-shape and Σ-shape CFS purlins subjected to geometrical and strength constraints provided in Eurocode 3 [7]. Liu et al. [8] used the Direct Strength Method (DSM) [9,10] and CUFSM and exploited Bayesian classification trees to find a cross section with largest nominal strength. Lee et al. [11,12] used a micro-genetic algorithm and optimized cold-formed steel channel beams under uniformly distributed loads as well as lipped channel columns under axial compression and proposed optimum design curves based on different geometrical parameters. In a direct follow-up to this work, Leng et al. [13] explored three optimization algorithms including steepest descent, genetic algorithm and simulated annealing to find cross-sections with maximum compressive strength for simply supported cold-formed steel columns of different lengths. Buckling loads were calculated using CUFSM and the nominal compressive strength was derived following DSM. A cross section with constant coil width was selected and discretized into equal length strips. The turn-angles between adjacent strip elements were used as design variables to find the optimal shape of the cross section. Except a geometrical crossing constraint, the design space was searched freely and the results were presented for long (16 ft) and short (4 ft) columns. Leng et al. has recently extended this work to optimization for major axis flexural strength as well as optimization under a number of manufacturing constraints including a limit on the number of rollers used to fold the cross section [14]. Gilbert et al. [15,16] have also proposed a self-shape optimization strategy where they adopt GA operators for the purpose of minimizing the cross section

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area (weigh) of simply supported CFS columns with different lengths.

Cold-formed steel members are usually thin-walled and have open cross sections. They are therefore prone to local, distortional, and global (Euler) buckling [1]. The goal of this work is to examine the role of boundary conditions on optimal column cross sections, i.e. cross sections that maximize compressive capacity of a column with a given length, coil width, and sheet thickness. This study is therefore an extension of the work in [13] in the sense that it covers different scenarios for the boundary conditions of the column. In addition, we discuss the choice of longitudinal basis functions in the context of FSM, and some computational strategies used to explore different instability modes of a given cross section. As far as the optimization algorithm is concerned, the available options are algorithms based on formal mathematical programming (e.g. steepest descent or gradient based algorithms), or algorithms based-on principles of stochastic search. Our computational experience (see also [13]) shows that gradient descent based solutions are highly sensitive to the initial design, but are more practical (e.g. symmetrical). The stochastic search algorithms, on the other hand, are computationally expensive but do a better job in exploring the design space while being relatively insensitive to the initial design. In this paper we choose a somewhat hybrid approach. We first start exploring the design space via a stochastic search algorithm. Genetic Algorithm (GA), a general-purpose, derivative-free, stochastic search algorithm is used here. To arrive at practical designs we put constraints on the geometrical properties of the optimal cross section. We finally refine the near-optimal folding of the cross section through a few steps of the gradient descent optimization.

The organization of this paper is as follows. In Section 2 the basics and formulation of FSM, including the choice of longitudinal shape functions, along with strategies used to identify the critical buckling modes and their associated buckling loads (beyond those appropriate for simply supported columns; see [9]) are discussed. Section 3 discusses how finding the optimal cross section is formulated in the form of an optimization problem and lays out the framework used to solve the problem. In Section 4 optimal cross sections for columns under uniform compression are provided and critically analyzed. Different column support conditions will be considered and the impact of the controlling stability mode on the optimal design and its strength are explored. Finally, Section 5 is devoted to concluding remarks.

2. Finite strip method (FSM) for general boundary conditions

2.1. Basics and formulation

FSM is a semi-analytical method, in which a (prismatic) thin-walled member is discretized into a number of predefined longitudinal narrow strips (see Fig. 1).

The interpolating functions in the transverse direction are assumed to be polynomials while judiciously chosen trigonometric functions are used in the longitudinal direction. The computational efficiency that results from circumventing the need for discretization in the longitudinal direction comes in handy in cases where repetitive analyses (such as those needed in an optimization context) must be performed. The other advantage is the easy (more straightforward) identification of instability modes which is an essential part of many design methodologies such as Direct Strength Method (DSM). We review here the very basics of FSM as applied in stability analysis of CFS members. The reader interested in more details is referred to the well-known text by Cheung and Tham [17], and numerous papers and technical reports in the literature (see [3] for example).

The displacement field vector within the finite strip, $\boldsymbol{u} = [u\ v\ w]^T$, is approximated in the form of the following series:

$$\boldsymbol{u} = \sum_{p=1}^{M} \boldsymbol{N^p d^p} \tag{1}$$

The shape function matrix and nodal displacement vector, expressed in terms of their uncoupled membrane and flexural parts, are as follows:

$$\boldsymbol{N^p} = \begin{bmatrix} \boldsymbol{N^p_{uv}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{N^p_{w\theta}} \end{bmatrix}, \ \boldsymbol{d^p} = \begin{bmatrix} \boldsymbol{d^p_{uv}} \\ \boldsymbol{d^p_{w\theta}} \end{bmatrix} \tag{2}$$

The components of the above matrices involve the product of the usual linear and cubic shape functions, and the pth longitudinal basis function satisfying the boundary condition in place, which is denoted by Y_p (see [4] for more details). Adopting Love's postulates [18], one assumes the following form for the strains:

$$\varepsilon = \varepsilon_{\boldsymbol{l}} + \varepsilon_{\boldsymbol{n}} \tag{3}$$

where ε_l and ε_n are linear and nonlinear components of strain respectively. These strains can be expressed as a summation of the membrane and flexural parts:

$$\varepsilon_{\boldsymbol{l}} = \varepsilon_{\boldsymbol{lm}} + \varepsilon_{\boldsymbol{lb}}, \varepsilon_{\boldsymbol{n}} = \varepsilon_{\boldsymbol{nm}} + \varepsilon_{\boldsymbol{nb}} \tag{4}$$

with subscripts $\boldsymbol{m}$ and $\boldsymbol{b}$ denoting membrane and flexural (bending) actions. Plugging Eq. (1) into (4) and substituting the results into the total potential energy functional:

$$\boldsymbol{\Pi} = \frac{1}{2}\int \boldsymbol{\sigma^T}\varepsilon_l dV - \int \boldsymbol{\sigma_0^T}\varepsilon_n dV \tag{5}$$

where $\boldsymbol{\sigma^T} = (\sigma_x, \sigma_y, \tau_{xy})$, $\boldsymbol{\sigma_0^T} = (0, -T/t, 0)$, and T is the constant edge traction (force per unit length, see Fig. 1), one can identify the elastic and geometric stiffness matrices for a finite strip. It turns out that the elements of these matrices are functions of the following integrals:

$$I_1 = \int_0^a Y_p Y_q dy, \quad I_2 = I_3 = \int_0^a Y_p^{''} Y_q dy,$$

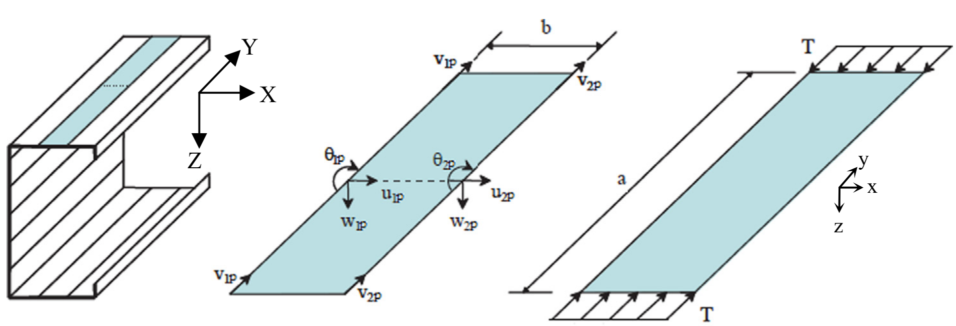

Fig. 1. Finite strip discretization, DOFs, local axes, and external edge traction for a strip.