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Elastic buckling of uniformly compressed thin-walled regular polygonal tubes

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ABSTRACT

This paper investigates the elastic buckling (bifurcation) behaviour of uniformly compressed thin-walled tubular members with single-cell regular polygonal cross-sections (RCPS), such as those employed to build transmission line structures, towers, antennas and masts. A specialisation of Generalised Beam Theory (GBT) for RCPS, reported in a recent paper (Gonçalves and Camotim, 2013) [1], is used to obtain both analytical and numerical results concerning the most relevant buckling modes and provide novel and broad conclusions on the structural behaviour of this type of members. In particular, local, cross-section extensional, distortional and multi-mode (including global flexural) buckling phenomena are addressed. For validation purposes, the GBT-based results are compared with solutions taken from the literature and also with numerical values obtained from finite strip analyses.

1. Introduction

Thin-walled tubular members with regular (equiangular and equilateral) convex polygonal cross-sections (RCPS—see Fig. 1 for geometry and notation) are widely employed in many areas of the engineering practice. Due to structural optimisation requirements, these members are often characterised by high wall width-to-thickness ratios and, therefore, are highly susceptible to complex buckling phenomena, which must be appropriately accounted for when assessing their resistance. The widespread use of RCPS members has naturally fostered a significant amount of investigation concerning their structural behaviour, but it is also true that information concerning the general elastic buckling (bifurcation) behaviour of these members is relatively scarce.

It may be stated that the local (plate-like) buckling behaviour of RCPS members is rather well-known. In fact, if the internal angle between consecutive walls is well below 180°, critical loads can be accurately predicted through the analysis of a single wall, simply supported along the lateral edges[1] (e.g., [2]). However, other buckling phenomena may be critical and, in that case, this simplified model may lead to non-negligible errors.

Several theoretical and numerical investigations concerning the buckling behaviour of RCPS have been carried out in the past. In [3], the local buckling of long polygonal tubes under combined uniform compression and torsion was investigated using stability functions and assuming that the line junctions between adjoining walls remain straight. Equilateral triangle and square sections were studied in detail and, in the first case, it was found that the buckling mode for pure compression is indeterminate, with a longitudinal nodal line appearing between the lateral edges of one wall, as first pointed out in [4]. In [5], extensive local buckling solutions for RCPS have been presented in the form of stability curves, for various cross-section wall numbers n. This study showed that the use of the classical plate buckling formula (E is Young's modulus and ν is Poisson's ratio)

$$\sigma_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2, \quad (1)$$

with the buckling coefficient $k=4$ (as for simply supported plates) leads to accurate results for most cases, although the triangular and pentagonal members exhibit significantly higher local buckling stresses. More recently, the local buckling of triangular (equilateral and isosceles) tubes was examined in [6] and analytical solutions were developed.

When the number of walls n increases, the internal angle between walls also increases and buckling may occur with non-null displacements along the wall junctions, as in the case of circular tubes. This phenomenon was experimentally investigated in [7], where it was concluded that, for uniformly compressed tubes, the transition between collapse modes occurs for internal angles greater than 160° (approximately). Similar conclusions were obtained in [8], on the basis of the numerical analysis of a

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[1] For RCPS with an even number of walls, the simply supported plate model leads to the exact solution.

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two-plate assembly, and more recently in [9], from experimental results.

The local buckling analysis of RCPS members with $n=4$–8, under axial compression or bending, was addressed in [10] using finite strip analyses. For uniformly compressed members, the results showed once again that odd n values lead to $k>4$, particularly for low n values. Moreover, for low plate slenderness values, a drop in the k value was observed, particularly for $n=5$ and $n=7$. For members under uniform bending, the results indicated that the k values increase by about 25%, due to the beneficial stress distribution around the cross-section. In this study, no displacements along the wall junctions were reported.

The present paper focuses on the elastic buckling (bifurcation) behaviour of RCPS members and aims at providing a general and novel mechanical insight into the problem, from both analytical and numerical perspectives. The approach adopted is based on a recently developed Generalised Beam Theory (GBT, e.g., [11–13]) specialisation for RCPS [1]. This specialisation (i) stems from the fact that n-sided RCPS exhibit rotational symmetry of order n and (ii) makes it possible to calculate more rationally and efficiently the cross-section orthogonal deformation mode sets, thus leading to accurate solutions with just a few modes.

Section 2 presents the GBT fundamental equations for the linear stability analysis of members under uniform compression and summarises the conclusions presented in [1] concerning the calculation of the various cross-section deformation modes. Section 3 is devoted to the investigation of single-mode buckling, namely local, cross-section extensional and distortional buckling. Moreover, the parameter range for which each buckling mode is critical is determined. Section 4 addresses multi-mode buckling, which requires the simultaneous consideration of all GBT deformation modes, including the global (flexural) and shear modes. Finally, the paper closes with some concluding remarks (Section 5).

Throughout the paper, results of several parametric and numeric studies are presented and discussed. For validation purposes, the GBT-based results are compared with available solutions and/or values obtained from finite strip analyses performed with CUFSM [14].

Concerning the notation, vectors and matrices are represented in bold letters. Partial derivatives are indicated by subscripts following a comma, e.g., $f_{,x}=\partial f/\partial x$. A virtual variation is denoted by δ. As previously mentioned, the cross-section geometric parameters are shown in Fig. 1, which also indicates the GBT cross-section discretisation employed in this work: n natural nodes and an arbitrary number m of equally spaced intermediate nodes in each wall (the total number of intermediate nodes thus equals $n\times m$). Moreover, the problems under analysis are generally written in terms of the non-dimensional geometric parameters

$$\beta_1=\frac{L}{r},\quad \beta_2=\frac{r}{t},\tag{2}$$

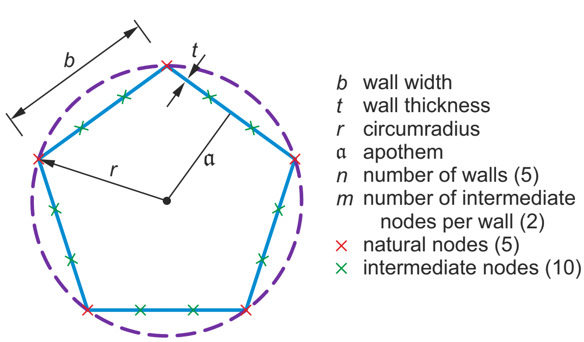

Fig. 1. Geometry and notation for regular convex polygonal sections (RCPS).

where L is the member length. Then, the plate width-to-thickness ratio and plate buckling formula (1) may be written as

$$\frac{b}{t}=2\beta_2\sin\left(\frac{\pi}{n}\right),\qquad \frac{\sigma_{cr}}{E}=\frac{k\pi^2}{48(1-\nu^2)\beta_2^2\sin^2(\pi/n)}.\tag{3}$$

2. GBT buckling analysis for RCPS

2.1. Fundamental equations

Following the usual GBT kinematic description and the notation adopted in [15], with the wall mid-surface local axes (x, y, z) shown in Fig. 2, the displacement field for each wall is given by

$$\boldsymbol{U}(x,y,z)=\begin{bmatrix}U_x\\U_y\\U_z\end{bmatrix}=\begin{bmatrix}u(x,y)-zw_{,x}(x,y)\\v(x,y)-zw_{,y}(x,y)\\w(x,y)\end{bmatrix},\tag{4}$$

where u, v, w are the mid-surface displacement components along x, y and z, respectively, expressed as

$$\begin{aligned}u(x,y)&=\sum_{k=1}^{D}\overline{u}_k(y)\phi_{k,x}(x),\\v(x,y)&=\sum_{k=1}^{D}\overline{v}_k(y)\phi_k(x),\\w(x,y)&=\sum_{k=1}^{D}\overline{w}_k(y)\phi_k(x),\end{aligned}\tag{5}$$

where $\overline{u}_k$, $\overline{v}_k$, $\overline{w}_k$ are the components of the $k=1,\ldots,D$ deformation modes (shape functions) and $\phi_k(x)$ are their amplitude functions along the member length, the problem unknowns.

For uniformly compressed members ($\sigma_{xx}=\lambda\overline{\sigma}$, where λ is the load parameter and $\overline{\sigma}$ is the reference stress value), the associated GBT equation system for linear stability analyses may be obtained from the linearisation of the virtual work equation, yielding (e.g., [15,16])

$$\int_L\begin{bmatrix}\delta\boldsymbol{\phi}\\\delta\boldsymbol{\phi}_{,x}\\\delta\boldsymbol{\phi}_{,xx}\end{bmatrix}^t\begin{bmatrix}\mathbf{B}&\mathbf{0}&\mathbf{D}_2\\\mathbf{0}&\mathbf{D}_1&\mathbf{0}\\(\mathbf{D}_2)^t&\mathbf{0}&\mathbf{C}\end{bmatrix}\begin{bmatrix}\boldsymbol{\phi}\\\boldsymbol{\phi}_{,x}\\\boldsymbol{\phi}_{,xx}\end{bmatrix}dx+\lambda\int_L\begin{bmatrix}\delta\boldsymbol{\phi}\\\delta\boldsymbol{\phi}_{,x}\\\delta\boldsymbol{\phi}_{,xx}\end{bmatrix}^t\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{X}_{\sigma}^{\overline{v}+\overline{w}}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{X}_{\sigma}^{\overline{u}}\end{bmatrix}\begin{bmatrix}\boldsymbol{\phi}\\\boldsymbol{\phi}_{,x}\\\boldsymbol{\phi}_{,xx}\end{bmatrix}dx=0,\tag{6}$$

where L is the member length, $\boldsymbol{\phi}$ is a vector containing the buckling mode amplitude functions $\phi_k(x)$, $\delta\boldsymbol{\phi}$ is the corresponding virtual variation and the GBT modal matrices are given by

$$\mathsf{B}_{ij}=\mathsf{B}_{ij}^M+\mathsf{B}_{ij}^B=\int_S\frac{E}{1-\nu^2}\left(t\overline{v}_{i,y}\overline{v}_{j,y}+\frac{t^3}{12}\overline{w}_{i,yy}\overline{w}_{j,yy}\right)dy,$$

$$\mathsf{C}_{ij}=\mathsf{C}_{ij}^M+\mathsf{C}_{ij}^B=\int_S\frac{E}{1-\nu^2}\left(t\overline{u}_i\overline{u}_j+\frac{t^3}{12}\overline{w}_i\overline{w}_j\right)dy,$$

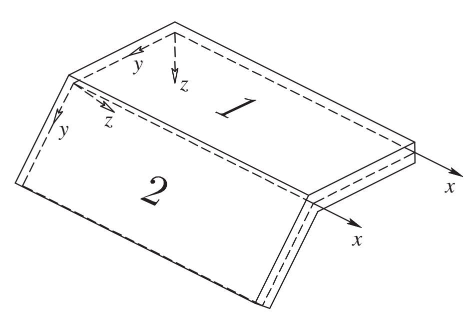

Fig. 2. Arbitrary thin-walled member local coordinate systems.