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Dynamic torsional buckling of cylindrical shells in Hamiltonian system

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ABSTRACT

By considering the effect of stress waves in a Hamiltonian system, this paper treats dynamic buckling of an elastic cylindrical shell which is subjected to an impact torsional load. A symplectic analytical approach is employed to convert the fundamental equations to the Hamiltonian canonical equations in dual variables. In a symplectic space, the critical torsion and buckling mode are reduced to solving the symplectic eigenvalue and eigensolution, respectively. The primary influence factors, such as the impact time, boundary conditions and thickness, are discussed in detail through some numerical examples. It is found that boundary conditions have limited influence except free boundary condition in the context of the scope in this paper. The localization of dynamic buckling patterns can be observed at the free end of the shell. The new analytical and numerical results serve as guidelines for safer designs of shell structures.

1. Introduction

Cylindrical shells have been widely used as one of the basic components in many types of engineering structures. To improve the structural reliability and safety, it is of great significance to clarify the dynamic stability of cylindrical shells under various impact loads. Although dynamic buckling of cylindrical shells under an axial impact has been studied extensively, by contrast, dynamic torsional buckling of cylindrical shells receives relatively little attention due to the inherent mathematical difficulty. In some early theoretical studies, only approximate solutions were obtained for some special cases, such as that by Leyko and Spryszynski [1] in which dynamic buckling of a cylindrical shell subjected to a time-dependent torsion was analyzed by using an approximate energy method. Subsequently, by using the energy criterion proposed by Ru and Wang [2], Wang et al. [3] investigated dynamic stability of a plastic cylindrical shell subjected to impact torsion. In this analysis, the rigid-plastic linear hardening mode was adopted and the critical impact velocity was discussed in detail. More recently, Sofiyev et al. [4–6] conducted similar research for some new high-performance materials. Galerkin's method combined with the Ritz type variation method or Lagrange–Hamilton type principle was applied to estimate the effect of configurations of constituent materials. To explore a different approach with respect to those published approximate methods, a general perturbation method was developed by Wang et al. [7] to study the impact torsional buckling for an elastic cylindrical shell with an arbitrary imperfection. The result showed that imperfect geometry significantly influences the static torsional buckling load. A brief review on the dynamic behavior of simple structures was reported by Jones [8] in 1989. A few excellent monographs which discuss the various aspects of dynamic bucking can be referred to Simitses [9] and Lindberg and Florence [10].

There exist some experimental studies in this regard [11,12]. A survey shows that increasing post-buckling deformation always begins to take place at an initial linear stage, especially for axial impact buckling of cylindrical shells [13,14]. Hence, it offers a good opportunity to capitalize this fact and to apply the small deformation theory for understanding some dynamic buckling phenomena. Accordingly, the study of impact torsional buckling of cylindrical shells can be converted to a bifurcation problem by studying the propagation of stress waves [15,16]. The buckling deformation in the disturbed region can be obtained based on the bifurcation buckling theory. Unfortunately, only some roughly theoretical analyses are conducted by authors and approximate solving methods are employed. Research works with rigorous analytical solutions to the titled problem for different boundary conditions have been limited.

For static torsional bucking, Yamaki and Kodama [17] presented some accurate solutions by directly integrating the fundamental equations and eight symmetric boundary conditions were treated in the study. However, the solution method cannot constitute a rigorous approach and it cannot provide a uniform analysis process. In short, most of the analytical methods available can be regarded as some variations of the Lagrangian system approach. The basic equations are expressed as some higher-order partial differential equations and they are usually solved by assuming some shape functions in one or two spatial dimensions. This approach is commonly known as the semi-inverse approach.

In an attempt to overcome the shortcomings of the semi-inverse method, Zhong [18] presented a revolutionary solution

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methodology, the symplectic analytical approach, for solving some basic problems in solid mechanics. Through the Legendre transformation, the governing equations are transformed into lower-order Hamiltonian canonical equations in dual variables. Later, Xu et al. [19] established a new symplectic system to investigate dynamic torsional buckling of clamped cylindrical shells. Nevertheless, the system is merely effective for a shell based on the Timoshenko's model and it is not applicable to treat local buckling problems. To substantiate the unknown research area, therefore, this paper develops a new symplectic system to analyze dynamic torsional buckling of cylindrical shells with various boundary conditions by considering the stress wave propagation based on Donnell's shell theory. Using numerical examples, some interesting insights into this problem are discussed in detail.

2. Basic equations

An elastic cylindrical shell with radius R, thickness h, length l, Young's modulus E, Poisson's ratio υ and material density ρ, subjected to an impact torque is illustrated in Fig. 1. Adopting a circular cylindrical coordinate system, the constitutive relations are

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \chi \end{Bmatrix}, \tag{1}$$

where $\boldsymbol{N}=\{N_x, N_\theta, N_{x\theta}\}^T$ and $\boldsymbol{M}=\{M_x, M_\theta, M_{x\theta}\}^T$ are the stress resultants and stress couples per unit length, respectively. The elastic coefficient matrixes are given by

$$\mathbf{E} = \frac{Eh}{1-\upsilon^2}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & (1-\upsilon)/2 \end{bmatrix}, \quad \mathbf{D} = \frac{Eh^3}{12(1-\upsilon^2)}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & 1-\upsilon \end{bmatrix}.$$

The geometric equations relating the strain vector $\boldsymbol{\varepsilon}=\{\varepsilon_x, \varepsilon_\theta, \varepsilon_{x\theta}\}^T$ and curvature vector $\boldsymbol{\chi}=\{\kappa_x, \kappa_\theta, \kappa_{x\theta}\}^T$ with the displacements are given by

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_\theta = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} - w\right), \quad \varepsilon_{x\theta} = \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x},$$
$$\kappa_x = -\frac{\partial^2 w}{\partial x^2}, \quad \kappa_\theta = -\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2}, \quad \kappa_{x\theta} = -\frac{1}{R}\frac{\partial^2 w}{\partial x \partial \theta}. \tag{2}$$

Introducing a stress function F, we have

$$N_x = \frac{1}{R^2}\frac{\partial^2 \mathrm{F}}{\partial \theta^2}, \quad N_\theta = \frac{\partial^2 \mathrm{F}}{\partial x^2}, \quad N_{x\theta} = -\frac{1}{R}\frac{\partial^2 \mathrm{F}}{\partial x \partial \theta}. \tag{3}$$

The Lagrange functional, which is dependent on the incremental displacements (u, v, w) and stress function F, consists of the elastic potential energy, potential energy due to external load and kinetic energy as [19]

$$\begin{aligned} \tilde{L} &= \Pi_\varepsilon + \Pi_w - V_t \\ &= \int_{t_0}^{t_1} dt \int_0^{2\pi} R\, d\theta \int_0^l \Bigg[\frac{1}{R^2}\frac{\partial^2 \mathrm{F}}{\partial \theta^2}\frac{\partial u}{\partial x} + \frac{\partial^2 \mathrm{F}}{\partial x^2}\frac{1}{R}\left(\frac{\partial v}{\partial \theta} - w\right) \\ &\quad - \frac{1}{R}\frac{\partial^2 \mathrm{F}}{\partial x \partial \theta}\left(\frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}\right) - \frac{1}{2Eh}\left(\left(\frac{\partial^2 \mathrm{F}}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 \mathrm{F}}{\partial \theta^2}\right)^2 \right. \\ &\quad \left. -2(1+\upsilon)\left(\frac{\partial^2 \mathrm{F}}{\partial x^2}\frac{1}{R^2}\frac{\partial^2 \mathrm{F}}{\partial \theta^2} - \left(\frac{1}{R}\frac{\partial^2 \mathrm{F}}{\partial x \partial \theta}\right)^2\right)\right) \\ &\quad + \frac{1}{2}D\left(\left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2}\right)^2 - 2(1-\upsilon)\left(\frac{\partial^2 w}{\partial x^2}\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2} - \left(\frac{1}{R}\frac{\partial^2 w}{\partial x \partial \theta}\right)^2\right)\right) \\ &\quad - \frac{N_{x\theta}{}^0}{R}\frac{\partial w}{\partial x}\frac{\partial w}{\partial \theta} - \frac{1}{2}\rho h\left(\frac{\partial u}{\partial t}\right)^2 - \frac{1}{2}\rho h\left(\frac{\partial v}{\partial t}\right)^2 - \frac{1}{2}\rho h\left(\frac{\partial w}{\partial t}\right)^2 \Bigg] dx, \end{aligned} \tag{4}$$

where $D = Eh^3/[12(1-\upsilon^2)]$ and t is the time. Then, the Lagrange density function can be obtained through the variational principle and integration by parts as

$$\begin{aligned} L &= \frac{1}{R^2}\frac{\partial^2 \mathrm{F}}{\partial \theta^2}\frac{\partial u}{\partial x} + \frac{\partial^2 \mathrm{F}}{\partial x^2}\frac{1}{R}\left(\frac{\partial v}{\partial \theta} - w\right) - \frac{1}{R}\frac{\partial^2 \mathrm{F}}{\partial x \partial \theta}\left(\frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}\right) \\ &\quad - \frac{1}{2Eh}\left(\frac{\partial^2 \mathrm{F}}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 \mathrm{F}}{\partial \theta^2}\right)^2 + \frac{1}{2}D\left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2}\right)^2 - \frac{N_{x\theta}{}^0}{R}\frac{\partial w}{\partial x}\frac{\partial w}{\partial \theta} \\ &\quad - \frac{1}{2}\rho h\left(\frac{\partial u}{\partial t}\right)^2 - \frac{1}{2}\rho h\left(\frac{\partial v}{\partial t}\right)^2 - \frac{1}{2}\rho h\left(\frac{\partial w}{\partial t}\right)^2, \end{aligned} \tag{5}$$

where $N_{x\theta}{}^0 = \tilde{T}_{x\theta}/\left(2\pi R^2\right)$ is the torsion stress resultant of the pre-buckling membrane state. Using the Hamiltonian principle $\delta\tilde{L} = 0$, the governing equations for this Donnell's shell theory can be obtained as

$$\begin{cases} \frac{\delta \tilde{L}}{\delta \mathrm{F}} = \frac{1}{Eh}\nabla^4 \mathrm{F} + \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0, \\ \frac{\delta \tilde{L}}{\delta w} = D\nabla^4 w - \frac{1}{R}\frac{\partial^2 \mathrm{F}}{\partial x^2} + \frac{2N_{x\theta}{}^0}{R}\frac{\partial^2 w}{\partial x \partial \theta} + \rho h \frac{\partial^2 w}{\partial t^2} = 0, \end{cases} \tag{6}$$

where $\nabla^2 = (\partial^2/\partial x^2) + \left(1/R^2\right)\left(\partial^2/\partial \theta^2\right)$ is the Laplacian operator.

3. Hamiltonian system and dual equations

For simplicity, the following dimensionless terms are defined:

$$X = \frac{x}{R}, \quad U = \frac{u}{R}, \quad V = \frac{v}{R}, \quad W = \frac{w}{R}, \quad \Phi = \frac{\mathrm{F}}{Eh^3}, \quad L = \frac{l}{R}, \quad H = \frac{h}{R},$$
$$\alpha = 12(1-\upsilon^2), \quad \beta = \alpha H^2, \quad T_{cr} = \frac{N_{x\theta}{}^0 R^2}{D}, \quad T = \frac{ct}{R}, \quad \gamma = \frac{\rho h c^2 R^2}{D}, \tag{7}$$

where $c = \sqrt{E/[2\rho(1+\upsilon)]}$ is the wave velocity [19]. By defining $\dot{W} = \partial W/\partial\theta$ and $\partial_X W = \partial W/\partial X$, further taking $\xi = -\dot{W}$ and $\psi = -\dot{\Phi}$, the dimensionless Lagrange density function can be expressed as

$$L = -\alpha W \partial_X^2 \Phi - \frac{1}{2}\beta\left(\partial_X^2 \Phi + \ddot{\Phi}\right)^2 + \frac{1}{2}\left(\partial_X^2 W + \ddot{W}\right)^2 - T_{cr}\dot{W}\partial_X W - \frac{1}{2}\gamma(\partial_T W)^2. \tag{8}$$

Introducing a vector $\boldsymbol{q} = \{q_1, q_2, q_3, q_4\}^T = \{W, \xi, \Phi, \psi\}^T$, the corresponding dual vector is given by

$$\mathbf{p} = \frac{\delta L}{\delta \dot{\mathbf{q}}} = \begin{Bmatrix} -\left(\dddot{W} + \partial_X^2 \dot{W}\right) - T_{cr}\partial_X W \\ -\left(\ddot{W} + \partial_X^2 W\right) \\ \beta\left(\dddot{\Phi} + \partial_X^2 \dot{\Phi}\right) \\ \beta\left(\ddot{\Phi} + \partial_X^2 \Phi\right) \end{Bmatrix}. \tag{9}$$

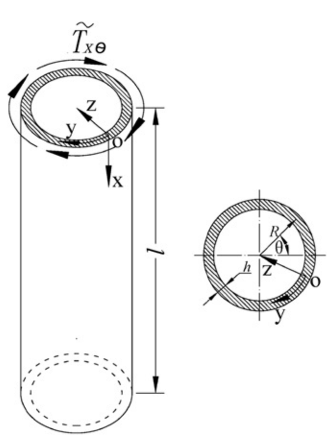

Fig. 1. Geometry for a cylindrical shell subject to an impact torque.