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Initial postbuckling behavior of thin-walled frames under mode interaction

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ABSTRACT

An L frame made up by beam and column having channel cross sections, has been analyzed in a previous work by two of the authors [14]. Depending on the aspect ratio and the joint configuration, it has been proved that the structure can exhibit two simultaneous buckling modes. Here using the asymptotic theory of elastic bifurcation that takes into account mode interaction, the initial slope of the bifurcated paths has been determined. Three cases of joint configurations, which are the more common used in welded connections, have been considered. For each case three admissible bifurcated paths have been found. Two of them show a slope having the same order of magnitude of the ones found in the absence of mode interaction while the remaining exhibits a slope largely steepest. Selecting, for each joint case, the bifurcated path with the higher slope and between them the smallest one, it is found that it is associated to the path which bifurcates at the higher critical load. This means that the stiffer structure is also the less imperfection sensitive. Finally for each one of the cases studied, the effect of initial imperfection has been considered and the real load carrying capacity of the frames has been determined. Finally some results have been compared with those obtained using the FE code ABAQUS.

1. Introduction

In a recent paper [14] which henceforth will be referenced as [RV], the authors have studied the effects of warping on the postbuckling behavior of frames made up by thin walled beams with channel cross sections.

The analysis has been focused on the behavior of a family of L frames obtained by varying the ratio L_1/L_2—L_1,L_2 being the lengths of the beam and the column, respectively—for three different joint configurations.

It has been found that, when the ratio L_1/L_2 approaches a precise value—which depends on the joint configuration—the first two critical loads come to coincide.

Fig. 7 of [RV]—that is reported as Fig. 3 in this paper—shows that, in those circumstances, the initial slope of the bifurcated paths tends to increase very rapidly.

Now, as those results have been obtained by means of a *standard* bifurcation analysis—that is assuming that each buckling load has multiplicity $n=1$ and different buckling loads are sufficiently far one from another—they must be considered unreliable.

On the other hand it is well known that, when two (or more) critical loads coincide or are very close, correct results can be obtained by using an appropriate modification of the asymptotic method.

The aim of this paper is to refine the analysis performed in [RV] by carrying out an investigation on the behavior of the structure when the first two buckling loads coincide. This is done using the mode interaction theory proposed by Koiter [7] in the form in which as been recast by Budiansky [3].

The results obtained show that mode interaction can lead to a significant increase of the initial slope of the bifurcated paths, as usually happens in these circumstances (see e.g. [10,12,4]).

In order to assess how this slope is related to the imperfection sensitivity of the structure, the analysis has been widened considering the effect of small initial imperfections on the frame behavior.

The results obtained show that the occurrence of mode interaction cause a sensible erosion of the load carrying capacity of the structure that, in some cases, is found to be greater than 30%.

A brief discussion on the comparison between the results obtained by means of the asymptotic analysis and the ones found using the path-following method implemented in the FE code ABAQUS, is done.

2. A direct one-dimensional model for thin-walled beams

In this section we give a short account of the 1D model adopted in the analysis. For more details, the reader is referred to [RV] and [17].

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Let us consider a plane cross-section and denote by o and c its centroid and shear center, respectively. We can think to orthogonally attach a section to each point of a straight line of length ℓ, that we call the beam axis. In particular, we consider the cases in which the axis is the line of the centroids or, alternatively, the line of the shear centers. We fix orthogonal cartesian co-ordinates with x_1 parallel to the beam axes and a consistent ortho-normal right-handed vector basis ($\boldsymbol{i}_1,\boldsymbol{i}_2,\boldsymbol{i}_3$). Suitable strain measures [13,11] are

$$\begin{aligned}
&\mathbf{E} = \mathbf{R}^\top \mathbf{R}' = \chi_1 \boldsymbol{i}_2 \wedge \boldsymbol{i}_3 + \chi_2 \boldsymbol{i}_3 \wedge \boldsymbol{i}_1 + \chi_3 \boldsymbol{i}_1 \wedge \boldsymbol{i}_2,\\
&\mathbf{e}_o = \mathbf{R}^\top \mathbf{p}'_o - \mathbf{q}'_o = \varepsilon_1 \boldsymbol{i}_1 + \varepsilon_2 \boldsymbol{i}_2 + \varepsilon_3 \boldsymbol{i}_3,\\
&\mathbf{e}_c = \mathbf{R}^\top \mathbf{p}'_c - \mathbf{q}'_c = \mathbf{e}_o + \mathbf{E}\mathbf{c} = \varepsilon_{1c} \boldsymbol{i}_1 + \varepsilon_{2c} \boldsymbol{i}_2 + \varepsilon_{3c} \boldsymbol{i}_3\\
&\quad = (\varepsilon_1 + \chi_2 c_3 - \chi_3 c_2)\boldsymbol{i}_1 + (\varepsilon_2 - \chi_1 c_3)\boldsymbol{i}_2 + (\varepsilon_3 + \chi_1 c_2)\boldsymbol{i}_3,\\
&\alpha, \eta = \alpha',
\end{aligned} \tag{1}$$

where $\mathbf{c} = c - o = c_2 \boldsymbol{i}_2 + c_3 \boldsymbol{i}_3$; $\mathbf{p}_o(x_1,t)$, $\mathbf{p}_c(x_1,t)$ are the vector-valued functions describing the present placements of the axes given by $\mathbf{q}_o(x_1)$ and $\mathbf{q}_c(x_1)$ in the reference shape; $\mathbf{R}(x_1,t)$ is the proper orthogonal tensor-valued cross-sections rotation from the reference to the present shape; and $\alpha(x_1,t)$ is a scalar-valued function that we consider as a coarse descriptor of warping. Besides, χ_1 stands for the torsion curvature (twist) and χ_2, χ_3 for the bending curvatures; ε_1 is the elongation of the centroidal axis, ε_2, ε_3 are the shear strains between this axis and the cross-section planes; ε_{1c}, ε_{2c}, ε_{3c}, are the same quantities referred to the axis of the shear centers.

The displacement of the points belonging to the centroidal and shear center axes together with the rotation are given the following component form:

$$\begin{aligned}
\mathbf{u} &= \mathbf{p}_o - \mathbf{q}_o = u_1 \boldsymbol{i}_1 + u_2 \boldsymbol{i}_2 + u_3 \boldsymbol{i}_3\\
\mathbf{u}_c &= \mathbf{p}_c - \mathbf{q}_c = u_{1c} \boldsymbol{i}_1 + u_{2c} \boldsymbol{i}_2 + u_{3c} \boldsymbol{i}_3
\end{aligned} \quad \mathbf{R} = \mathbf{R}_3 \mathbf{R}_2 \mathbf{R}_1 \tag{2}$$

where $\mathbf{R}_1$ is a rotation of amplitude θ_1 around $\boldsymbol{i}_1$; $\mathbf{R}_2$ is a rotation of amplitude θ_2 around $\mathbf{R}_1 \boldsymbol{i}_2$; $\mathbf{R}_3$ is a rotation of amplitude θ_3 around $\mathbf{R}_2 \mathbf{R}_1 \boldsymbol{i}_3$.

By substituting (2) in (1) one obtains nonlinear strain–displacement relationships that we synthetically refer to in the form

$$\epsilon = e(u) \tag{3}$$

We assume that the beam is homogeneous, nonlinearly hyperelastic, and that its elastic energy density, φ is

$$\begin{aligned}
\varphi = {} & \tfrac{1}{2} a \varepsilon_1^2 + \tfrac{1}{2} g_2 \varepsilon_{2c}^2 + \tfrac{1}{2} g_3 \varepsilon_{3c}^2 + \tfrac{1}{2} c \chi_1^2 + \tfrac{1}{2} b_2 \chi_2^2 + \tfrac{1}{2} b_3 \chi_3^2 + \tfrac{1}{2} h \eta^2\\
& + \tfrac{1}{2} k (\alpha - \xi \chi_1)^2 + \tfrac{1}{2} (d \varepsilon_1 + f_2 \chi_2 + f_3 \chi_3 + g \eta) \chi_1^2
\end{aligned} \tag{4}$$

Using (1) we can make the derivative of φ with respect to the strain components, ε_{1c}, ε_{2c}, ε_{3c}, χ_1, χ_2, χ_3, α, η, obtaining the stress measures reported in (7) of [RV].

Here we want just to recall that with Q_1, Q_2, Q_3, we denote the normal and shear forces applied at the shear center, S_1, S_2, S_3, are the twisting couple and the bending torques, evaluated with respect to the shear center, τ and μ are the bishear and bimoment, respectively. It is also worth noting that the constitutive relationships obtained, keep into account the couplings between extension and torsion, bending and torsion, warping and torsion, respectively [21,9,22].

In this way the virtual work density of the stress, reads

$$\delta\varphi = \varphi' \delta\epsilon = Q_1 \delta\varepsilon_{1c} + Q_2 \delta\varepsilon_{2c} + Q_3 \delta\varepsilon_{3c} + S_1 \delta\chi_1 + S_2 \delta\chi_2 + S_3 \delta\chi_3 + \tau \delta\alpha + \mu \delta\eta \tag{5}$$

where the prime denotes derivative of each function with respect to its own argument. Now, by putting

$$\begin{aligned}
\mathbf{s} &= Q_1 \boldsymbol{i}_1 + Q_2 \boldsymbol{i}_2 + Q_3 \boldsymbol{i}_3\\
\mathbf{S} &= S_1 \boldsymbol{i}_2 \wedge \boldsymbol{i}_3 + S_2 \boldsymbol{i}_3 \wedge \boldsymbol{i}_1 + S_3 \boldsymbol{i}_1 \wedge \boldsymbol{i}_2
\end{aligned} \tag{6}$$

and using Eq. (5), we can write

$$\int \delta\varphi \, dx_1 = \int (\mathbf{s} \cdot \delta\mathbf{e}_c + \mathbf{S} \cdot \delta\mathbf{E}_c + \tau \delta\omega + \mu \delta\eta) \, dx_1 \tag{7}$$

which, when the variations are interpreted as spatial velocity fields, coincides with the expression of the (virtual) internal power (15) in [17]. This means that the equilibrium equations underlying the present formulation are the (18) and $(12)_5$ of [17].

3. Bifurcation analysis

Let us consider a system of hyperelastic beams acted upon by external conservative loads, whose total potential energy can be written in the form

$$\pi(u,\epsilon,\lambda) = \int (\varphi(e(u)) - \lambda u) \, dx_1 \tag{8}$$

λ being the load parameter.

The condition of equilibrium, obtained by requesting $\pi(u,e(u),\lambda)$ to be stationary, can be written as

$$\begin{aligned}
&\sigma\delta\epsilon - \lambda\delta u = \sigma e'(u)\delta u - \lambda \delta u = 0 \qquad \forall \delta u\\
&\sigma = \varphi'(\epsilon) = s(\epsilon)\\
&\epsilon = e(u)
\end{aligned} \tag{9}$$

where a prime stands for differentiation of a function with respect to its own argument.

Eq. (9), supplied with appropriate boundary conditions, gives a *nonlinear* boundary value problem whose solutions are the equilibrium states of the structure.

3.1. Asymptotic solution in the case of simultaneous buckling modes

Let us assume, now, that (9) admits at least two solution branches: $(u^f(\lambda),\lambda(t))$ and $(u^b(t),\lambda(t))$—t being a real parameter—that we call *fundamental* and *bifurcated*, respectively. In addition, we assume that the two branches intersect at a point where $t=0$, so that $\lambda(0) = \lambda_s$ and $u^f(\lambda_s) = u^b(0)$.

If the fundamental solution is known, we may introduce the *difference* fields

$$\begin{aligned}
&v(t) = u^b(t) - u^f(t)\\
&\varrho(t) = \sigma^b(t) - \sigma^f(t)\\
&\gamma(t) = \epsilon^b(t) - \epsilon^f(t)
\end{aligned} \tag{10}$$

and look for the asymptotic expansion of the bifurcated solution near the bifurcation point, that is

$$\begin{aligned}
&v(t) = \overline{v} t + \tfrac{1}{2} \overline{\overline{v}} t^2 + o(t^2)\\
&\lambda(t) = \lambda_s + \overline{\lambda} t + \tfrac{1}{2} \overline{\overline{\lambda}} t^2 + o(t^2)
\end{aligned} \tag{11}$$

where superimposed bars denote derivatives with respect to t evaluated at $t=0$.

In view of (10) and (11), all the fields in (9) can be seen as functions of the parameter t. The *nonlinear* BVP (9) can be transformed in a sequence of *linear* BVPs by means of the following procedure.

The first derivative of (9) with respect to t is

$$\begin{aligned}
&(\dot\sigma e'(u) + \sigma e''(u)\dot u - \dot\lambda)\delta u = 0 \quad \forall \delta u\\
&\dot\sigma = s'(\epsilon)\dot\epsilon\\
&\dot\epsilon = e'(u)\dot u
\end{aligned} \tag{12}$$

where a superimposed dot denotes derivative with respect to t.

As (12) holds true $\forall t$ and is satisfied for both the fundamental and the bifurcated paths, we can evaluate it for each path at $t=0$. Then, by making the difference between the latter and former