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Ultimate strength prediction by semi-analytical analysis of stiffened plates with various boundary conditions

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ABSTRACT

Semi-analytical elastic methods for stiffened plate analysis are computationally very efficient. In addition to eigenvalue analysis, such methods may also offer a viable approach for the prediction of ultimate strength limits (USLs) of the plates when combined with appropriate strength criteria. In this paper, existing strength criteria are discussed, and extended criteria proposed for plates with various stiffener arrangements and boundary conditions such as full out-of-plane supports along all edges or plates with a free or partially stiffened edge. The extended criteria reflect in a simplified manner the effect of redistribution of stresses due to the formation of local plastic regions at stiffeners, supporting edges and in the plate interior. The equilibrium path is traced using large deflection theory and the Rayleigh–Ritz approach on an incremental form. The approach is able to account for the reserve strength of slender plates in the postbuckling region. With the considered criteria included, good agreement is obtained with results from fully nonlinear finite element analyses for different support conditions and for a variety of plate and stiffener dimensions.

1. Introduction

Steel structures such as ships and orthotropic bridge decks consist of a large number of plate elements with different support conditions. Such plates, stiffened or unstiffened, may be able to carry loads beyond those causing yielding at the outer fibres, due to redistribution of plate stresses caused by formation of plastic regions. To account for such effects, it is necessary to consider both geometrical and material nonlinearity.

In general cases in which explicit strength formulas [1,2] are not readily applicable, due to complex geometries and boundary conditions, ultimate strengths can be computed using nonlinear finite element methods (FEM). However, due to the large number of elements that may have to be considered, nonlinear finite element analysis (FEA) is often impractical for routine design. Using FEM will also be quite time consuming, due to modeling and calculation time. In such cases, linear elastic, semi-analytical methods offer a viable approach, as they generally are computationally efficient when combined with appropriate strength criteria. Such approaches, in the framework of large deflection theory, are considered here.

The main objective of the present study is to investigate the applicability of various strength criteria that may be incorporated into semi-analytical methods for application to in-plane loaded plates that may have different stiffener arrangements and different support conditions along the plate edges. The considered plate "types" are chosen due to their practical importance. They are rectangular, and may be fully supported in the out-of-plane direction, by simple or rotationally restrained supports at all edges, or they may have one free, or partially stiffened, edge. The major focus is on the latter plate types in this study, which represents a continuation of a previous study on fully supported plates [3]. The applicability of the proposed criteria will be verified by comparison with strength results from fully nonlinear FEA for a number of plate cases.

2. Plate mechanics, simplified methods and failure modes

For plates with predominantly in-plane loads, membrane stresses are redistributed due to out-of-plane displacements from the softer interior regions of the plates to the stiffer regions. Such redistribution will also be affected by possible yielding. The stiffest parts of the

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plate are normally at the edges, and possibly at the stiffeners in local bending cases. It is these stiff parts that are critical for the strength of the plates. When the capacity of these parts are exhausted, no more stresses can be redistributed, and additional in-plane loading cannot be applied without causing total plate collapse as the ultimate strength is reached. Due to such stress redistributions, the ultimate strength may significantly exceed the elastic plate buckling load, and the load giving first yield.

In rigorous semi-analytical elastic methods, geometrical nonlinearity is accounted for by including large deflection formulations. On the other hand, material nonlinearity, due to the formation of local plastic regions, is accounted for in a simplified manner. In one such approach, Paik and Lee [4] treated progressing plasticity numerically by removing material in plastic regions of simply supported, rectangular plates. Other, and probably computationally more efficient approaches, are given for instance in Byklum [5], Brubak et al. [6], Brubak and Hellesland [7,8], Paik et al. [9], and Cheng et al. [10]. To capture limit loads (ultimate load capacity) in these studies, a simplified strength criterion, defined by the von Mises' yield criterion, is used. Applied to membrane stresses, this criterion allows indirectly for some yielding to take place since the stresses at outer fibres may exceed the membrane stress (at the midplane).

In the latter work [8] it was concluded, for the simply supported rectangular plates considered, that the membrane stress criterion may become non-conservative for cases where bending stresses are important. This may for instance be the case for local bending of rather thick plates with irregular stiffener arrangements, for which the bending stresses at critical parts become significant. In addition, this criterion does not account for plastic regions in the stiffeners for global bending (buckling) cases. Brubak and Hellesland [3] proposed improved strength criteria that included stiffener yielding, which led to very good strength predictions for arbitrarily stiffened, fully supported plates inn local or global bending.

A case of practical importance, which has not been considered before in a strength context, is that of plates with a free or flexibly (partially) supported edge. Brubak and Hellesland [11] developed a semi-analytical pre- and postbuckling analysis for such plates, but only with superficial treatment of strength aspects [12]. Such plates will exhibit considerably larger displacements than plates supported along all edges. These displacements are particularly pronounced at or near a free edge, or somewhat away from a partially supported edge. As a consequence, considerable bending stresses may form at this edge, or in the interior of such plates, and cause plate yielding and thereby an additional redistribution of stresses to the stiffer, critical parts at supporting edges and stiffeners. Such yielding was not adequately reflected by the previously proposed criteria [3]. In order to also capture this effect, there is a need for alternative, improved criteria in order to achieve reliable results. In this paper, an effort is made to propose criteria that cover a wider range of cases.

In order to establish reliable strength criteria, it is of importance to identify the different failure modes of the plate. Possible failure modes, briefly discussed above, are related to plate and stiffener geometry, boundary condition and external loads. Typical cases for plates subjected to in-plane axial stresses are summarized as follows:

- *Unstiffened, simply supported plate*: failure initiated by stresses at the edges reaching the material yield stress.
- *Stiffened, simply supported plate*: failure due to plate yielding at the stiffener locations or at the stiff edges, and formation of plastic regions in the stiffeners in cases with global bending modes.
- *Supported, partially or fully rotationally restrained (clamped) plate*: failure initiated by yielding at the plate edges.
- *Unstiffened plate with a free edge*: failure due to yielding at the supporting edges, caused in part by redistribution of membrane stresses and in part by redistribution due to large bending stresses (and yielding) in the interior of the plate.
- *Stiffened plate with a free or partially supported edge*: failure due to yielding at the supporting edges, caused in part by redistribution of membrane stresses and in part by redistribution due to large bending stresses (and yielding) in the stiffeners and the interior of the plate.

3. Proposed strength criteria

3.1. Typical plates

A plate is usually a part of a larger structure surrounded by adjacent girders and plates, and its boundaries are typically supported with strong longitudinal and transverse girders or strong flanges preventing out-of-plane displacements. Such plates can be considered fully supported in the out-of-plane direction, either by simple supports or with additional rotational restraints along the edges to reflect rotational continuity with neighbouring plates, and it may be subjected to stresses at all edges, as illustrated in Fig. 1(a). In other cases, such as girder webs, one of the edges may be free or supported by a flexible structure, as illustrated in Fig. 2. The proposed criteria will cover plates of both kinds. Plate details will be given in conjunction with the actual numerical comparisons.

3.2. Fully out-of-plane supported edges

In strength analyses in other studies [3,8], the authors found, as mentioned previously, that predicted strengths using the von Mises' yield criterion for the membrane stresses can be non-conservative in cases where the bending stresses are important. In particular, this was found to be the case for (1) stiffened plates

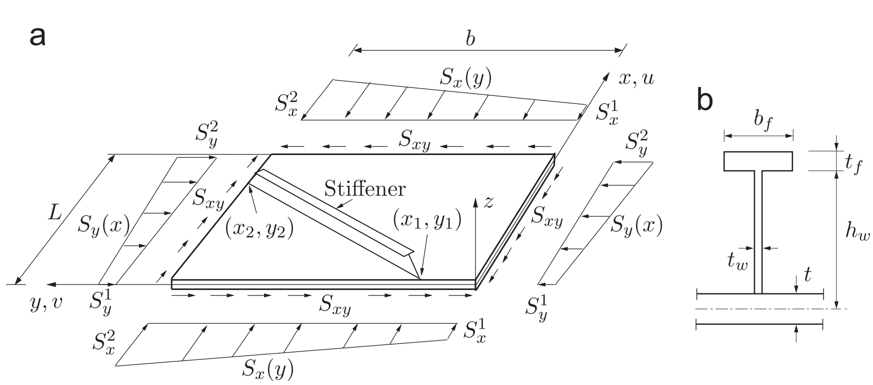

Fig. 1. (a) Stiffened plate subjected to in-plane shear stress and in-plane, linear varying compression or tension stress, and (b) cross-section of an eccentric stiffener.