Thin-Walled Structures 62 (2013) 53–64

Contents lists available at SciVerse ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Vibration analysis of a Mindlin elastic plate under a moving mass excitation by eigenfunction expansion method

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ARTICLE INFO

Article history:
Received 9 January 2012
Received in revised form
9 July 2012
Accepted 13 July 2012
Available online 23 September 2012

Keywords:
Mindlin plate
Moving mass
Moving force
Eigenfunction expansion method
Dynamic response

ABSTRACT

Elastodynamic response of an undamped moderately thick plate, with arbitrary boundary conditions, under a moving mass is investigated. The FSDT (first-order shear deformation plate theory or Mindlin plate theory) is selected as the governing equations of motion. By using direct separation of variables and eigenfunction expansion method, the three basic variables defining the displacement field in FSDT, are transformed into a series including the eigenfunctions of plate free vibration with time dependent amplitude factors. By neglecting the inertia interaction between mass and the plate, the closed-form solution is derived while it remarkably reduces the complication of numerical computations. Having the moving mass inertia effect taken into account as well as all the convective terms of its out-of-plane acceleration components, a semi analytical solution is presented. The most interested moving mass trajectories in engineering application of the issue, orbiting and rectilinear paths are investigated in numerical examples and the results for a simply supported rectangular Mindlin plate are obtained. The method introduced is not limited by shape of the plate and trajectory of the moving mass. Concentrated moving loads as well as other arbitrarily selected distribution-area of loads are covered in the formulations. Parametric survey is carried out by using both FSDT and CPT (classical plate theory or Kirchhoff plate theory) and remarkable differences between CPT and FSDT modeling results, for moderately thick plates, emphasize the significance of using FSDT. For thin plates, the FSDT yields closely near the same results as that of CPT which demonstrates the generality of the solutions presented in this article with regard to capability of capturing the under study plate dynamics for a wider range of the plate thickness with appropriate precision.

1. Introduction

Evaluating the dynamic response of solid structures, excited by moving loads is a practical engineering problem and has attracted the attention of numerous researchers [1]. Investigating the performance of the media which undergoes moving loads is significant for the engineers, for instance, in design of safe and reliable structural systems. Nowadays, the ever increasing interest in economic construction and earthquake resistance systems imply the demand for lighter weight structures. Consequently, the effect of relatively large masses traveling on such structures should be explored with enough accuracy. Railways and bridges are some typical examples in transportation industry. Their dynamic behavior under moving mass of traversing vehicles and trains should be scrutinized to perform a rational safe design of mentioned structures [2]. The slab type bridges [3] and deck of ships on which the aircrafts land, can be regarded as plate elements supporting moving loads. In mechanical engineering, high speed precision machinery processes involve the problem of a structure under the influence of moving mass [4].

Extensive research works on the topic of a solid media excited by moving loads, have been devoted to the cables [5,6] and beams [7–14] in which the related mathematical model is of less complexity in comparison with that of plates. Namely, Bilello et al. [7] studied an experimental model of a single-span bridge loaded by a moving mass traveling on it. The results of their experimental small-scale prototype and those obtained by eigenfunction expansion method showed a good agreement. Yavari et al. [8] have introduced discrete element technique (DET), to study the dynamics of Timoshenko beams. They presented the capability of DET to capture the dynamic response of both thin and thick beams due to a moving concentrated mass. Nikkhoo et al. [9] studied dynamic behavior and modal control of an Euler–Bernoulli

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http://dx.doi.org/10.1016/j.tws.2012.07.014

beam under excitation of a lumped moving mass. Once they limited the inertia interaction to the vertical component of the moving mass acceleration alone. Subsequently, they concluded that the resulting approximate solution outputs coincide with that of exact formulation for velocities less than a so called "critical velocity". They pointed out the fact that moving mass velocity rarely exceeds this limit in practice. Euler–Bernoulli beam under distributed moving mass was studied by Esmailzadeh and Ghorashi [10]. The effect of mass inertia and its length of distribution were discussed in their work. In a different investigation, Siddiqi and Golnaraghi [11] have assessed the dynamics of a flexible cantilever beam carrying a moving mass-spring. They evaluated inertial resonance behavior for special parametric conditions. Kiani et al. [12] presented a comprehensive study on the effects of different beam theories as well as its boundary conditions on the dynamic behavior of beams under a moving mass excitation via reproducing kernel particle method (RKPM). They proved if the slenderness of the base beam is less than some certain limits, only the higher-order beams theory would lead to trustful results. Dynamics of multispan thin and shear deformable viscoelastic beams under a moving load and moving mass have been scrutinized by Kiani et al. [13,14] via a numerical scheme named as the generalized moving least square method (GMLSM). They stated the inertia effect would become so crucial as the beam span number increases. Moreover, their results indicated that the possibility of separation between the moving mass and the base beam during the course of vibration increases if the values of velocity and mass weight of the load as well as the beam span number, increase. However, by increasing the relaxation rate of the beam material, this possibility would decrease as well.

Cifuentes and Lalapet article [4] is an instance of researches done in accordance with moving mass exciting a plate. The article investigated the transverse deflection time-history of a rectangular thin plate by employing CPT. Furthermore, finite element technique was selected to evaluate plate center point dynamic response under a moving lumped mass. They adopted the FEM mesh for an orbiting path of the moving mass. They disregarded the convective components of moving mass out-of-plane acceleration terms. Consequently, the importance of considering the inertia interaction of plate and moving mass was underlined. Shadnam et al. [15] performed a study on dynamic response of a simply supported rectangular thin plate under the influence of a concentrated moving mass traversing on an arbitrary trajectory. They employed eigenfunction expansion method as well as CPT as the governing equation of plate dynamics. They only took the vertical component of the out-of-plane moving mass acceleration term into account. Rofooei and Nikkhoo [16] carried out a study on vibration control of thin rectangular plates by active piezoelectric patches. A concentrated moving mass was assumed as the excitation term and Kirchhoff plate theory was considered as the governing equation of plate. Their results were obtained by means of eigenfunction expansion method. Similar methods were used in another research work [17] to achieve an extensive parametric study on the maximum deflection of a rectangular simply supported thin plate under a concentrated moving mass and either rectilinear and orbiting trajectories were included. In both recently mentioned works [16,17], the full term components of moving mass out-of-plane acceleration were considered to detect the exact solutions. They have also compared the results of the exact and approximate solution in accordance with moving mass acceleration terms. As a result, they pointed out the approximate solution results in which only the vertical component of moving mass acceleration is taken into account is not of validity for moving mass velocities higher than a certain limit.

The researchers have employed different approaches to deal with the problem of solids vibration due to moving mass excitation with regard to load inertia consideration. Many solutions have been carried out by modeling moving mass via moving force neglecting the inertia interaction of moving mass and its supporting structure [1]. While some number of studies have highlighted the significance of moving mass inertia effect on dynamics of corresponding supporting structures, for high speeds of moving mass and large amounts of moving mass weight [4,17]. It is clear that the analytical modeling which regards the most realistic first assumptions yields results of more validity.

In this paper, the PDEs set of FSDT are employed as the constitutive equations of motion without applying any limitation to the plate boundary conditions. The plate is loaded under moving mass traversing on the plate surface. It is assumed that moving mass is traveling on an arbitrary trajectory and a general format of load distribution pattern on the plate surface is considered. By employing eigenfunction expansion method, the governing coupled PDEs of motion are ultimately reduced to a set of ODEs in the time domain.

Procedure of solution involves two major points of view. Once the inertia interaction of moving load and plate can be neglected, which causes more simplicity but less validity. In this regard, mathematical function defining the external load will only consist of gravitational weight of the load which acts as the moving force and the closed-form solution of before mentioned ODEs is derived. The real problem is also considered by taking the inertia interaction of mass and plate into account, which increases the complexity of the related mathematical problem and ultimately leads to a set of coupled ODEs to investigate moving load in the form of moving mass. For this condition, a semi-analytical approach is provided as a solution. In addition, the full terms of the mass out-of-plane acceleration components are considered. Numerical results are provided for the cases of SSSS, SCSC, CCCC and SCSF boundary conditions. The obtained results from extensive parametric studies for SSSS boundary condition have highlighted the importance of regarding mass inertia for high velocities and large mass values.

General format of the load distribution area is considered and solved in numerical examples by using FSDT for limiting cases of moving concentrated load, moving line load and moving load distributed on a rectangular zone. The results for both of CPT and FSDT are discussed in the form of parametric surveys for a simply supported rectangular plate as well with respect to variation of plate thickness. The two most interested trajectories in practice, rectilinear and orbiting paths are investigated widely. Provided parametric studies, finally shows a close agreement of CPT and FSDT corresponding outputs for thin plate. However, considerable differences of mentioned solutions for higher plate thicknesses rationalize the application of FSDT instead of CPT.

2. Problem definition, assumptions and solutions

Dynamics of an undamped moderately thick plate under a moving load is assumed. All the next coming discussions for the plates under study are based on linear, uniform and elastic plate material. Furthermore, the moving mass travels on the surface of the plate keeping the full contact condition during the whole course of movement.

All the related formulation and solution procedure for CPT which used in this research are achieved by means of eigenfuncion expansion method [16,17].

The constitutive coupled PDEs of FSDT in dynamics, considering the plate rotary inertia with assuming frictionless contact of moving mass and plate surface, are as below [18]

$$\kappa Gh\left(\frac{\partial \beta_1}{\partial x}-\frac{\partial^2 W}{\partial x^2}+\frac{\partial \beta_2}{\partial y}-\frac{\partial^2 W}{\partial y^2}\right)+\rho h\frac{\partial^2 W}{\partial t^2}=P(x,y,t) \quad (1)$$