Thin-Walled Structures 62 (2013) 131–141

Contents lists available at SciVerse ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Buckling analysis of nonhomogeneous orthotropic thin-walled truncated conical shells in large deformation

Sofiyev A.H.[a,*], Kuruoglu N.[b]

[a] Department of Civil Engineering of Suleyman Demirel University, Isparta 32260, Turkey
[b] Department of Mathematics and Computer Science, Faculty of Arts and Sciences, Bahcesehir University, Istanbul, Turkey

ARTICLE INFO

Article history:
Received 26 October 2011
Received in revised form
9 August 2012
Accepted 9 August 2012
Available online 28 September 2012

Keywords:
Non-homogeneity
Orthotropic materials
Truncated conical shell
Large deformation
Non-linear buckling

ABSTRACT

In this study, the buckling analysis of homogeneous and non-homogeneous orthotropic, thin walled truncated conical shells under axial load and in large deformation has been investigated. First, the governing relations are derived using the large deformation theory with von Karman–Donnell-type of kinematic non-linearity. Then modified Donnell type stability and compatibility equations of non-homogeneous orthotropic thin-walled truncated conical shells in large deformation are obtained and solved analytically. Finally, influences of the non-homogeneity, orthotropy and the variation of the shell geometry on the non-linear axial buckling load are investigated. Comparing the results of this study with those in the literature validates the present analysis.

1. Introduction

As a common structure, the homogeneous and non-homogeneous composite truncated conical shell has been widely applied in many fields such as space flight, rocketry, aviation, nuclear reactors, jet nozzles, and such other civil, chemical, mechanical, submarine and aerospace engineering technology, etc. The buckling analyses of such shells are very important for their applications, and have been of considerable research interest in recent years. The basic information for the buckling behavior of composite shells was described and summarized by Reddy [1]. The fundamental concepts on the present subject—such as geometric non-linearity, bifurcation, and limit loads were interpreted by Brush and Almroth [2], Agamirov [3] and Amabili [4]. The buckling behavior of the orthotropic conical shell in small deformation was first studied by Singer [5] and then a number of investigations have considered the mechanical behavior of orthotropic conical shells [6–10].

In the study of the stability of a thin-walled conical shell the prebuckling deformation may be of the order of the thickness. Thus, a non-linear theory is probably necessary for such investigations. There is much literature on the buckling analysis of homogeneous isotropic shells in large deformation [11–16]. A review of the literature shows that few studies have been carried out to investigate the buckling and vibration of homogeneous orthotropic shells in large deformation [17–23].

In recent years, new types of composite materials have been used in engineering and many investigations consider non-homogeneous orthotropic materials. In various technological situations are demanding that the non-homogeneity of orthotropic materials should be taken into account for the buckling behavior of structural elements. The non-homogeneity of the materials stems from the effects of humidity, surface and thermal polishing processes and methods of production, which render the physical properties of materials, vary from point to point (random, piecewise continuous or continuous functions of coordinates). Furthermore, certain parts of structural elements have to operate under radiation and elevated temperatures and which cause non-homogeneity in the material, i.e., the constants of the material become functions of space variables. When non-homogeneous materials deform, they retain their shapes up to the point of rupture. Hence, in the computations of structural members made of such materials, the fundamental relations and governing equations of deformable body mechanics are applicable [24–27]. Published literatures on the analysis of composite orthotropic structures with variable material properties are limited in number [28–38]. All these solutions are based on the small deflection theory.

Previous studies show that geometric non-linearity plays a significant role in the buckling behavior of homogeneous shells. As the geometrical non-linearity is taken into account in the governing equations of non-homogeneous shells, unpredictable behaviors may be occur. Therefore, it is of vital importance to

* Corresponding author. Tel.: +90 246 2111195; fax: +90 246 2370859.
E-mail address: asofiyev@mmf.sdu.edu.tr (A.H. Sofiyev).

0263-8231/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.tws.2012.08.002

study the non-linear response of non-homogeneous materials [39–46]. All of these studies focus on the non-linear buckling behavior of functionally graded shells.

To date, the non-linear buckling behavior of homogeneous and non-homogeneous orthotropic truncated conical shells has not been discussed because of difficulties due to non-linear mathematics and complicated structures. The current research deals with the non-linear buckling behavior of homogeneous and non-homogeneous orthotropic truncated conical shells subjected to uniform axial load, when the Young's moduli continuously through the thickness coordinate direction.

2. Governing equations

As shown in Fig. 1 a thin non-homogeneous orthotropic truncated conical shell subjected to a uniform axial load, T, is considered. The structure is referred to a curvilinear coordinate system (S, θ, ζ), where S and θ axes lie along the generator and in the circumferential direction on the reference surface of the cone, respectively, and the ζ axis, being perpendicular to the plane of the first two axes, lies in the inwards normal direction of the cone, R_1 and R_2 indicate the radii of the cone at its small and large ends, respectively, γ denotes the semi-vertex angle of the cone. H is height, L is the length and h is the thickness of the truncated cone. S_1 and S_2 are the distances from the vertex to the small and large bases, respectively. Also, u, v and w denote displacement (due to loads) of a point in the middle surface in the direction of a generator, the circumferential direction, and the inward normal direction, respectively. The axes of orthotropy are parallel to the curvilinear coordinates S and θ.

The non-homogeneity of the material of the shell is assumed to arise due to the variation of Young's moduli along the thickness direction ζ as (Lomakin [24]; Babich and Khoroshun [31]; Sofiyev and Schnack [33]).

$$\left[E_S\left(\bar{\zeta}\right), E_\theta\left(\bar{\zeta}\right), G_0\left(\bar{\zeta}\right)\right] = \varphi_1\left(\bar{\zeta}\right)[E_{0S}, E_{0\theta}, G_0]; \quad \bar{\zeta} = \zeta/h \tag{1}$$

where E_{0s} and $E_{0\theta}$ are the Young's moduli in S and θ directions, respectively, and G_0 is the shear modulus on the plane. Additionally, $\varphi_1\left(\bar{\zeta}\right) = 1 + \mu\varphi\left(\bar{\zeta}\right)$, where $\varphi\left(\bar{\zeta}\right)$ is the continuous function of the non-homogeneity defining the variation of the Young's moduli, satisfying the condition $\left|\varphi\left(\bar{\zeta}\right)\right| \le 1$, and μ is a non-homogeneity coefficient, satisfying $0 \le \mu \le 1$.

In this study, the non-homogeneity function of the material of the truncated conical shell is assumed to be power function which, $\varphi\left(\bar{\zeta}\right) = \pm\bar{\zeta}^q$, $q = 1, 2, 3, \ldots$. (Khoroshun and Koshevoi [25]; Lal [34]; Sofiyev et al. [38])

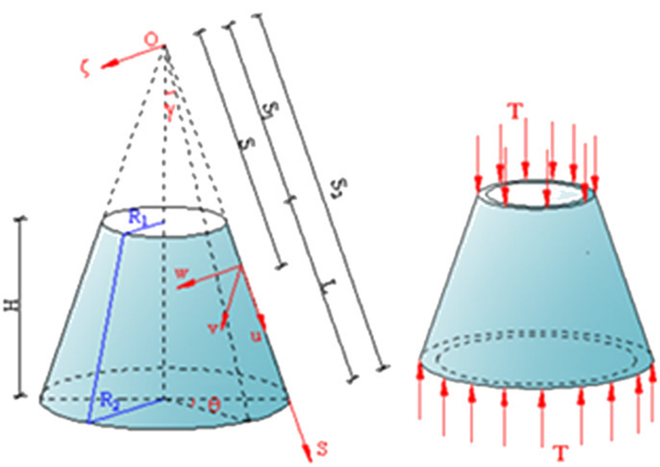

Fig. 1. Geometry of a truncated conical shell under axial load.

According to von Karman non-linear strain–displacement relations, the strain components on the middle plane of truncated conical shells are

$$\begin{pmatrix} e_S \\ e_\theta \\ 2e_{S\theta} \end{pmatrix} = \begin{bmatrix} \frac{\partial u}{\partial S} + \frac{1}{2}\left(\frac{\partial w}{\partial S}\right)^2 \\ \frac{1}{S}\frac{\partial v}{\partial \theta_1} + \frac{u}{S} - \frac{w\cot\gamma}{S} + \frac{1}{2S^2}\left(\frac{\partial w}{\partial \theta_1}\right)^2 \\ \frac{1}{S}\frac{\partial u}{\partial \theta_1} + \frac{\partial v}{\partial S} - \frac{v}{S} + \frac{1}{S}\left(\frac{\partial w}{\partial S}\frac{\partial w}{\partial \theta_1}\right)^2 \end{bmatrix} \tag{2}$$

where e_S and e_θ are the normal strains in the curvilinear coordinate directions S and θ on the reference surface, respectively, $e_{S\theta}$ is the shear strain and $\theta_1 = \theta\sin\gamma$.

According to the flexural shell theory, the stress-displacement relations for non-homogeneous orthotropic truncated conical shells are given as follows:

$$\begin{bmatrix} \sigma_S \\ \sigma_\theta \\ \sigma_{S\theta} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} e_S - \zeta\frac{\partial^2 w}{\partial S^2} \\ e_\theta - \zeta\left(\frac{1}{S^2}\frac{\partial^2 w}{\partial \theta_1{}^2} + \frac{1}{S}\frac{\partial w}{\partial S}\right) \\ e_{S\theta} - \zeta\left(\frac{1}{S}\frac{\partial^2 w}{\partial S \partial \theta_1} - \frac{1}{S^2}\frac{\partial w}{\partial \theta_1}\right) \end{bmatrix} \tag{3}$$

where the quantities Q_{ij}, $\quad i,j = 1, 2, 6$, are

$$Q_{11} = \frac{E_{0S}\varphi_1\left(\bar{\zeta}\right)}{1 - \nu_{S\theta}\nu_{\theta S}}, \quad Q_{22} = \frac{E_{0\theta}\varphi_1\left(\bar{\zeta}\right)}{1 - \nu_{S\theta}\nu_{\theta S}},$$
$$Q_{12} = Q_{21} = \nu_{\theta S}Q_{11} = \nu_{S\theta}Q_{22}, \quad Q_{66} = 2G_0\varphi_1\left(\bar{\zeta}\right) \tag{4}$$

in which $\nu_{S\theta}$ and $\nu_{\theta S}$ are the Poisson's ratios, assumed to be constant and satisfying $\nu_{\theta S}E_{0S} = \nu_{S\theta}E_{0\theta}$ (Grigorenko and Vasilenko [26]; Sofiyev and Schnack [33]).

The well-known force and moment resultants are expressed by (Reddy [1]):

$$[(N_S, N_\theta, N_{S\theta}), (M_S, M_\theta, M_{S\theta})] = \int_{-h/2}^{h/2} (\sigma_S, \sigma_\theta, \sigma_{S\theta})[1, \zeta]\, d\zeta \tag{5}$$

The relations between the force resultants and the stress function, Ψ, are as follows:

$$(N_S, N_\theta, N_{S\theta}) = \left(\frac{1}{S^2}\frac{\partial^2\Psi}{\partial\theta_1^2} + \frac{1}{S}\frac{\partial\Psi}{\partial S}, \quad \frac{\partial^2\Psi}{\partial S^2}, \quad -\frac{1}{S}\frac{\partial^2\Psi}{\partial S\partial\theta_1} + \frac{1}{S^2}\frac{\partial\Psi}{\partial\theta_1}\right) \tag{6}$$

By using large deformation theory of the thin-walled truncated shell, the stability and strain compatibility equations of truncated conical shells are given as follows (Agamirov [3]):

$$\frac{\partial N_S}{\partial S} + \frac{\partial N_{S\theta}}{\partial \theta_1} + \frac{N_S - N_\theta}{S} = 0 \tag{7}$$

$$\frac{\partial N_{S\theta}}{\partial S} + \frac{1}{S}\frac{\partial N_\theta}{\partial \theta_1} + \frac{2N_{S\theta}}{S} = 0 \tag{8}$$

$$\begin{aligned} &\frac{\partial^2 M_S}{\partial S^2} + \frac{2}{S}\frac{\partial M_S}{\partial S} + \frac{2}{S}\frac{\partial^2 M_{S\theta}}{\partial S\partial\theta_1} - \frac{1}{S}\frac{\partial M_\theta}{\partial S} + \frac{2}{S^2}\frac{\partial M_{S\theta}}{\partial\theta_1} + \frac{1}{S^2}\frac{\partial^2 M_\theta}{\partial\theta_1^2} + \frac{\cot\gamma}{S}N_\theta \\ &\quad + N_S\frac{\partial^2 w}{\partial S^2} - \frac{N_\theta}{S}\left(\frac{1}{S}\frac{\partial^2 w}{\partial\theta_1^2} + \frac{\partial w}{\partial S}\right) - 2N_{S\theta}\left(\frac{1}{S}\frac{\partial^2 w}{\partial S\partial\theta_1} - \frac{1}{S^2}\frac{\partial w}{\partial\theta_1}\right) = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} &\frac{\cot\gamma}{S}\frac{\partial^2 w}{\partial S^2} - \frac{2}{S}\frac{\partial^2 e_{S\theta}}{\partial S\partial\theta_1} - \frac{2}{S^2}\frac{\partial e_{S\theta}}{\partial\theta_1} + \frac{\partial^2 e_\theta}{\partial S^2} + \frac{1}{S^2}\frac{\partial^2 e_S}{\partial\theta_1^2} + \frac{2}{S}\frac{\partial e_\theta}{\partial S} - \frac{1}{S}\frac{\partial e_S}{\partial S} \\ &= \frac{1}{S^4}\left(\frac{\partial w}{\partial\theta_1}\right)^2 - \frac{2}{S^3}\frac{\partial w}{\partial\theta_1}\frac{\partial^2 w}{\partial S\partial\theta_1} - \frac{1}{S^2}\left[\frac{\partial^2 w}{\partial S^2}\frac{\partial^2 w}{\partial\theta_1^2} - \left(\frac{\partial^2 w}{\partial S\partial\theta_1}\right)^2\right] - \frac{1}{S}\frac{\partial w}{\partial S}\frac{\partial^2 w}{\partial S^2} \end{aligned} \tag{10}$$

Substituting expressions (3) in (5) and considering the resulting expressions together with relations (6), after some rearrangements the relations found for moments and strains, being substituted in (7)–(10), then for the simplicity of the