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A distortional semi-discretized thin-walled beam element

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ABSTRACT

Due to the increased consumption of thin-walled structural elements there has been increasing focus and need for more detailed calculations as well as development of new approaches. In this paper a thin-walled beam element including distortion of the cross section is formulated. The formulation is based on a generalized beam theory (GBT), in which the classic Vlasov beam theory for analysis of open and closed thin-walled cross sections is generalized by including distortional displacements. The beam element formulation utilizes a semi-discretization approach in which the cross section is discretized into wall elements and the analytical solutions of the related GBT beam equations are used as displacement functions in the axial direction. Thus the beam element contains the semi-analytical solutions. In three related papers the authors have recently presented the semi-discretization approach and the analytical solution of the beam equations of GBT. In this approach a full set of deformation modes corresponding to the homogeneous GBT equations are found. The deformation modes of which some are complex decouple the state space equations corresponding to the reduced order differential equations of GBT and allow the determination of the analytical solutions. Solutions of the non-homogeneous GBT differential equations and the distortional buckling equations have also been found and analyzed. Thus, these related papers are not dealing with an element but dealing with analytical solutions to the coupled differential equations.

To handle arbitrary boundary conditions as well as the possibility of adding concentrated loads as nodal loads the formulation of a beam element is needed. This paper presents the formulation of such a generalized one-dimensional semi-discretized thin-walled beam element including distortional contributions. It should be noticed that we are only dealing with a basic generalized beam theory and not an extended finite element formulation of an approximate beam element, which allows the addition of special (transverse extension and shear lag) modes. Illustrative examples showing the validity and the accuracy of the developed distortional semi-discretized thin-walled beam element are given and it is shown how the novel approach provides accurate results making it a good alternative to the traditional and time consuming FE calculations.

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1. Introduction

In the civil, mechanical and aerospace industry thin-walled members are often used because of the high strength and the effective use of material. Due to the increased consumption of thin-walled structural elements there has been increasing focus and need for more detailed calculations. Thus, it has been necessary to extend the classic beam theory to include the distortion of the cross section. Such an extension of the theory is considered in this paper and in a number of companion papers published by the authors [1–3], where a novel approach to generalized beam theory is formulated. A variety of other formulations and methods taking distortional displacements into account have been proposed for analysis of both open and closed cross sections. Thus, concerning analysis of thin-walled members including distortion of the cross section there are a number of methods available among which are: (i) the use of shell finite elements in the finite element method (FEM) [4,5], perhaps with utilization of recursive substructuring [6], (ii) the finite strip method (FSM) [7–11], and (iii) the use of approximate GBT-finite beam elements. Concerning approximate GBT-finite beam elements, specially the traditional first generation of generalized beam theory, known as GBT, initially proposed by Schardt in 1966 [12], has been very popular and fostered a lot of research and developments, mostly undertaken by a few independently working European groups, among others by Schardt [13], Davies [14], Lepistö [15], Simões da Silva and Simão [16], Gonçalves et al. [17], Gonçalves and Camotim [18] and Camotim and Silvestre [19].

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Silvestre and Camotim also extended the theory to include orthotropic materials, see [20,21], and experimental verifications have been presented by for example Rendek and Balaz [22]. Furthermore, Silvestre presents buckling solutions as well as non-linear post buckling solutions in [19]. For an overview and information about the research and development of GBT, see Camotim et al. [23,24].

The present novel approach to Generalized Beam Theory (GBT) involves a cross-section semi-discretization process as well as a determination of the natural cross-section eigenmodes and related axial solution functions by exact analytical solution of the related first-order GBT equations. Hereby the approach is different from the traditional GBT formulation developed by Schardt [12,13]. When Schardt uses the GBT equations to find distortional deformation modes the shear coupling stiffness terms are neglected. This corresponds to modal analysis with orthogonal (Rayleigh) damping in dynamic structural analysis. The solution of the shear coupled GBT equations for closed cross-sections was published by Hanf only in his thesis [25]. For closed (single or multi celled hollow) thin-walled cross-sections Schardt shows in his presentation of GBT [13] that the theory needs a relaxation of the Vlasov assumption of negligible shear strain in order to include the warping deformation associated with the "Bredt's shear flow" around each cell. However, it complicates the solution of the conventional GBT equations by introducing non-negligible shear coupling terms (off diagonal) in the GBT equations as can be seen in recent GBT formulations for closed thin-walled cross-sections, e.g. [26,17,27]. The present formulation therefore adheres to the definition of the warping function given by Kollbrunner and Hajdin [28], which adds the integral of the shear flow strains, see also [29–31].

The present GBT formulation for thin walled beams with both open and closed (single or multi cell) cross-sections can be regarded as an extension of classical Vlasov thin-walled beam theory to include distortional deformation modes as well as constant shear flows in the walls of the cross-section, see [32,28,33]. The innovative theoretical developments performed by introducing semi-discretization leads to a formulation, in which the rotational degrees of freedom are included, thus including local plate modes in the formulation even for the simplest discretization. It makes it possible to analyze thin-walled members with cross-section distortion and local plate behavior in a one-dimensional formulation through the linear combination of pre-established modes of deformation. In contrast to and as a considerable advance on the traditional GBT formulation this novel finite element based semi-discretization approach to generalized beam theory (GBT) solves the fourth-order differential equations to obtain the distortional displacements for a linear beam analysis. This means that we find the analytical solution to the differential GBT equations which through the magnitude of the eigenvalues gives a much better knowledge of the length scales of the modes. This also means that we find the exact mode shapes and amplitude solutions of the reduced order GBT equations related to the discretized cross section. In contrast, the conventional GBT formulation solves the equations using the approximate engineering methods, in which the shear coupling terms are neglected, producing orthogonal axial and transverse normal stress modes. In such a case the differential equations are not solved but a weak solution may be found through the introduction of the established approximate mode shapes and use of approximate modal amplitude functions. Thus the conventional GBT formulations use the produced approximated modes as shape functions in a virtual work or potential energy formulation leading to approximated finite GBT beam elements and the discretization has to be performed without proper prior knowledge of the problem length scales of the individual modes.

With respect to buckling the first application of the first generation of GBT to buckling analysis was published in 1970 by Schardt [34]. Among others also Davies [35], Simão [16] and Camotim [36] have investigated the area. Buckling analysis using GBT beam elements is an alternative to the use of finite-strip methods (FSM), see [37]. However, GBT is as its name states essentially a beam theory, whereas FSM essentially is based on plate theory. Therefore, FSM does not contain a natural decomposition into basic beam, distortional, local and other modes. Furthermore, conventional GBT does not contain other modes as mentioned above. Since the modal decomposition may lead to advantages in design of thin-walled structures using FSM a great deal of work has been performed by Ádány and Schafer to develop a constrained finite strip method (cFSM) and modal decomposition methods for open (single-branched) cross-sections, see [38–40]. The modal approaches of extended conventional GBT and cFSM formulations have been compared in [41]. The present novel developed semi-discretization approach to Generalized Beam Theory (GBT) is extended in [3] to include the geometrical stiffness terms which are needed for column buckling analysis and identification of buckling modes.

When cross sections distort, it means that they change shape. Distortional displacements can be of a local character in which the length scale is typically equal to or less than the cross section dimension or it can be non-local in which case the length scale is typically several times the cross section dimension or even longer. In the recent buckling literature and especially in codes there is a tendency, with respect to buckling, to distinguish between these two behaviors as distortional buckling and local buckling. In [1–3] we are operating with global, distortional non-local and distortional local modes when we define first-order displacement modes. However, in paper [3] which concerns buckling we have chosen to distinguish between distortional buckling and local buckling as in the recent codes and literature.

It should be noticed that shear deformation accommodating Bredt's shear flow around closed cells is included in the theory through the specific definition of the warping function, see Ref. [28]. Since we are dealing with a basic generalized beam theory and not an extended finite element formulation of an approximate beam element it makes sense to neglect the overall transverse shear deformation as in conventional beams. It is also important to note that shear lag is not included and that it would not be included even by including shear deformation as described by Kollbrunner and Hajdin [28]. Thus, we are only dealing with a beam element adhering to generalized beam theory and not an extended weak formulation of a finite beam element that allows the addition of special (transverse extension and shear lag) modes.

Let us introduce the contents of the following sections and illuminate the development. In the theories of beams, the displacements assumed are typically separated into a sum of displacement fields. In the sections involving the determination of such a displacement field, only one of these displacement fields is considered in the variational formulation. The basic kinematic assumptions of one of these displacement fields are introduced in Section 2. The displacements are separated into the product of cross-section displacement functions and the axial variation functions. Following this, the strain fields are derived. In Section 3 constitutive energy assumptions lead to the formulation of the internal and external elastic energy potential. In Section 4 the cross-section is discretized by straight wall elements in which the local transverse displacements, the warping displacements and the loads are interpolated. The element interpolation functions are introduced and the total elastic potential energy (for a single mode) is formulated in a semi-discretized form. To get a formulation resembling a generalization of Vlasov beam theory [32], Section 5 first briefly describes three main steps leading to the

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