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Nonlinear dynamics of rotating box FGM beams using nonlinear normal modes

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ABSTRACT

In this work an analysis is performed on the nonlinear planar vibrations of a functionally graded beam subjected to a combined thermal and harmonic transverse load in the presence of internal resonance. Adopting the direct perturbation MMS technique, the partial differential equations of motion of the beam are reduced to sets of first-order nonlinear modulation equations in terms of the complex modes of the beam. The assumption of steady-state values of centrifugal loads is evaluated. It has to be said that there is a lack of information about modeling of rotating box beams made of functionally graded materials (FGMs) under thermo-mechanical loads. The influence of the transverse load amplitude and the internal detuning parameter on the strength of nonlinear modal interaction is illustrated. It is also shown that the system exhibits periodic and quasiperiodic responses for a typical range of parameter values.

1. Introduction

The problem of modeling and studding of a rotating flexible beam has received a constant research interest in connection with the applications like flexible robot arms, rotor blades and spacecraft with flexible appendages. The most simplified representation of a rotating beam is a one-dimensional Euler–Bernoulli model. A uniform rotating beam of doubly symmetric cross-section is a special case. Owing to the stiffening effect of the centrifugal tension, one generally can expect the natural frequencies to increase with an increase in the speed of rotation. Several works have studied a rotating cantilever beam and approximate methods such as Rayleigh–Ritz, Galerkin, finite element methods, etc., has been used to find the natural frequencies [1–4]. However, the internally resonant analysis of rotating beams is rather rare in the literature [5–8]. Systematic procedures have been developed to obtain reduced-order models (ROMs) via nonlinear normal modes (NNMs) that are based on invariant manifolds in the state space of nonlinear systems [9–12]. These procedures initially used asymptotic series to approximate the geometry of the invariant manifold. They have been used by Pesheck et al. [5] to study the nonlinear rotating Euler–Bernoulli beam. Also, Pesheck et al. [6] employed a numerically-based Galerkin approach to generate accurate reduced-order models for large-amplitude, strongly non-linear motions.

Apiwattanalunggarn et al. [7] presented a nonlinear one-dimensional finite-element model representing the axial and transverse motions of a cantilevered rotating beam, which is reduced to a single nonlinear normal mode using invariant manifold techniques. They used this approach to study the dynamic characteristics of the finite element model over a wide range of vibration amplitudes. It is interesting to note that the interest of most works about nonlinear dynamic of rotating beams have focused on the reduced-order model as the invariant manifold solution. Turhan and Bulut [8] investigated the in-plane nonlinear vibrations of a rotating beam via single- and two-degree-of-freedom models obtained through a Galerkin discretization. They performed a perturbation analyses on single- and two-degree-of-freedom models to obtain amplitude dependent natural frequencies and frequency responses.

On the other hand, there are many papers concerning mechanics of beams made of FGMs. In the opinion of the authors, the articles presented in [13–15] among others offer interesting features, applications and calculation methodologies. These models are developed by means of different constitutive hypotheses (graded metallic-ceramic, graded multilayered, etc.) and displacement formulation (i.e. elementary Bernoulli–Euler or Timoshenko or Higher order shear deformable theories). The constitutive modeling is commonly related to a classical rule of mixtures and the material properties vary according to a power law expression [14] or an exponential expression [13].

Now, taking into account the technological context, it is important to mention that there is a lack of information about rotating beams constructed with FGMs. Piovan and Sampaio [16]

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introduced a nonlinear model for planar analysis of rotating beams with material properties graded along the solid cross-section. Only a few authors explored in recent years the dynamics of thin-walled beams constructed with functionally graded materials. Piovan and Sampaio [17] developed a theory to study the dynamics of telescopic thin walled beams made of FGMs. Oh et al. [18,19] introduced a couple of first-order-shear models to study vibratory patterns of spinning and rotating thin-walled beams constructed with FGMs. The papers given by Fazelzadeh et al. [20] and Fazelzadeh and Hosseini [21] also deal with rotating beams made of FGMs. However in these formulations no geometrical stiffness has been taken into account. The interest of these papers has been focused in the thermoelastic effects related to graded properties.

From the review of literature, it is found that the study of internal resonance in the area of cantilever rotating slender beam subjected to a harmonic transverse load has not yet been explored so far, neither in the context of composite materials nor in the context of functionally graded materials. The nonlinear modal interaction or the internal resonance in the system arising out of commensurable relationships of frequencies, in presence of parametric excitation due to periodic load can have possible influence on system behavior, which needs to be studied.

In the present paper, the nonlinear planar vibration of a rotating FGMs cantilever beam is analyzed, considering the dynamic condition of internal resonance. The model is based on a one-dimensional Euler–Bernoulli formulation where the geometric cubic nonlinear terms (due to midline stretching of the beam) are included in the equation of motion. The linear frequencies of the system are dependent on the rotation speed; this effect is used to activate the internal resonance. For a particular rotation speed the second natural frequency is approximately three times the first natural frequency and hence the first and second modes may interact due to a three-one internal resonance. Principal parametric resonance of first mode considering internal resonance is also analyzed. For a comprehensive review of nonlinear modal interactions, we refer the reader to [22–24]. The method of multiple scales (MMS) is used to attack directly the governing nonlinear partial differential equation of motion of the beam and reduce the problem to sets of first-order nonlinear modulation equations in terms of the complex modes of the beam [25]. These modulation equations are numerically analyzed for stability and bifurcations of trivial and nontrivial solutions. Bifurcation diagrams representing system responses with variation of parameters like amplitude and frequency of the lateral excitation load, frequency detuning of internal resonances and damping are computed with the help of a continuation algorithm [26]. The trivial state stability plots are presented and the modulation equations are numerically integrated to obtain the dynamic solutions (periodic, quasiperiodic and chaotic responses) for typical system parameters.

For the principal parametric resonance of first mode, the influence of internal resonance is illustrated in the frequency and amplitude responses. The system is shown to have Hopf and saddle node bifurcations for different parameter values. The influence of intensity of transverse load amplitude and frequency detuning for internal resonance on the strength of nonlinear modal interaction is illustrated. The system exhibits dynamic solutions like periodic and quasiperiodic responses for typical range of parameter values.

2. Funcionally graded material and its thermal properties

The laws of variation of the material properties along the wall thickness can be prescribed in order to bear in mind for different types of material gradation such as metal to ceramic or metal to metal (e.g. steel and aluminum). In this case, a simple gradation based on a power-law is employed. The law of variation of the elastic and mass properties along the wall-thickness e is:

$$P(n) = P_M + (P_C - P_M)\left(\frac{2n+e}{2e}\right)^K \tag{1}$$

where P(n) denotes a typical material property (i.e., density ρ or Young's modulus E or Poisson coefficient ν). Sub-indexes C and M define the properties of the material of the outer surface (normally ceramic) and inner surface (normally metallic), respectively. The exponent K, which is connected to the ratio of constituents in volume, can have different values that may vary between zero (i.e., a full ceramic phase) or infinity (i.e., a full metallic phase).

It is assumed that the beam is subjected to a steady-state one dimensional (1-D) temperature distribution through its thickness. The steady-state 1-D heat transfer equation is expressed by:

$$\frac{\mathrm{d}}{\mathrm{d}n}\left[k(n)\frac{\mathrm{d}T}{\mathrm{d}n}\right] = 0 \tag{2}$$

where k is the coefficient of the thermal conduction. The boundary conditions are:

$$T = T_M \text{ at } n = -\frac{e}{2} \text{ and } T = T_C \text{ at } n = \frac{e}{2} \tag{3}$$

The solution of Eq. (2) can be obtained by means of the polynomial series. Therefore, $T(n)$ is calculated as [27]:

$$T(n) = T_M + \frac{\Delta T}{\eta}\sum_{j=0}^{\psi}(-1)^j\frac{(k_C - k_M)^j}{(1+jK)k_M^j}\left(\frac{n}{e}+\frac{1}{2}\right)^{(1+jK)} \tag{4}$$

with

$$\eta = \sum_{j=0}^{\psi}(-1)^j\frac{(k_C - k_M)^j}{(1+jK)k_M^j} \tag{5}$$

where normally the upper limit of the summation is $\Psi \to \infty$, however by means of an elemental numerical study one can prove that Eq. (4) may be finely approximated by taking just a few terms, or more practically, $\Psi \geq 5$ as it was done by many researchers [28].

Throughout the numerical simulation T_M is taken 300 K. It is assumed that the properties of the FGMs are temperature-dependent and vary according to a law obtained experimentally. These are expressed in a general form as [19,29]:

$$p(n) = p_0(p_{-1}/T + 1 + p_1 T + p_2 T^2 + p_3 T^3) \tag{6}$$

in which p is a temperature-varying material property in general (i.e. modulus of elasticity, or Poisson's coefficient, etc.), T is the absolute temperature [°K] and the coefficient p_i is unique for a particular material and obtained by means of a curve fitting procedure. Thus the material properties can be represented as a function of the thickness and the temperature. It is clear that p_0 is the typical material property in absence of thermal effects.

3. Non-linear equations of motion

In this section the nonlinear equations of motion of a rotating box beam subjected to harmonic transverse loads are presented. The structural model of a thin-walled beam is shown in Fig. 1. The origin of the beam coordinate system (x, y, z) is located at the blade root at an offset R_0 from a rotation axis fixed in space. R_0 denotes the radius of the hub (considered to be rigid) in which the blade or beam is mounted. The hub rotates about its polar axis through the origin 0. We assume that the motion is planar and the cross sections remains plane during transverse bending. A doubly symmetric cross-section box-beam is used, thus uncoupling the out-of-plane (flapping) and in-plane (lead-lag) vibration.

Following the mathematical formulation developed by the authors in [30], considering in this case a Bernoulli–Euler theory