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Strain hardening in *M–P* interaction for metallic beam of I-section

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ABSTRACT

This paper derives bending moment–axial force (*M–P*) interaction relations for mild steel I-section by considering elastic–plastic and strain hardening idealisations with linear and parabolic strain hardening characteristics. The interaction curves have thus been developed. The interaction relations can predict strains, which is not possible in a rigid, perfectly plastic idealisation. The relations are obtained for different practically possible cases related to different locations of neutral axis. Two I-sections—one from DIN (German), IPE200 normally used for beams and another from AISC, W8 × 31 normally used for columns were considered for studying the characteristics of interaction curves.

1. Introduction

The impact loading on beams produces an inelastic response with visible damage or even fracture. Standard static methods of analysis with dynamic magnification factors, for example, are not adequate in such situations. The employment of numerical techniques, such as the finite element method may also not be of great help because of the problems of convergence in reaching up to failure due to the involvement of large displacements and strains.

The elastic and plastic interaction for different stress resultants involving different sections such as solid and hollow rectangular sections [1–3], circular sections [4], H and I sections [5–8], angle sections [9,10], and general hollow sections [11,12] has been studied extensively. Some extensive reviews of these research works may be found in several publications [3,13–15]. The elastic–plastic methods are currently adopted in modern standard codes of design to estimate the ultimate resistance of some steel structures, since they allow the beneficial effects of yielding in the redistribution of stresses to be taken into account. However, these simplified methods are not adequate for the failure analysis of different structural members. There are limited studies available for the interaction studies up to the ultimate capacity which are mostly experimental or numerical [16,17].

Alves and Jones [18,19] employed the continuum damage mechanics approach for predicting the static and dynamic failure of metallic beams but the method requires the values of several parameters, some of which are difficult to obtain. Some studies [20–22] are devoted to the theoretical anomalous dynamic response of beams and plates due to a short pulse loading for which elastic–plastic model was employed. The rigid perfectly plastic analysis [23] has been employed for the predictions of an elastic–plastic material [24,25]. However, a rigid, perfectly plastic analysis does not predict strains so that it is difficult to study failure unless some assumptions are made to overcome this difficulty [26].

In the present paper, kinematically admissible interaction relations for the simultaneous action of bending moment and an axial force on the I-sections have been developed for elastic–plastic and strain-hardening material idealisations. The relations are obtained for different practically possible cases related to different locations of neutral axis. The interaction curves developed for I-section may be easily degenerated to rectangular section. The development of interaction relations requires only the results from a standard uniaxial tensile test on the material. Two I-sections—one from DIN (German), IPE200 normally used for beams and another from AISC, W8 × 31 normally used for columns were considered for studying the characteristics of interaction curves.

2. Stress–strain diagram

The stress–strain diagram for mild steel is idealised as bilinear for small strains, whereas, two models – linear and parabolic – are used for strain-hardening (Fig. 1). Direct tensile test results for a mild steel specimen 't036' [19] are shown in Fig. 1. Thus, there

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Nomenclature

B	Width of beam
b	Thickness of web of the beam
H	Depth of beam
h	Thickness of flange
H_1	Distance of neutral axis (N.A.) from extreme compression fibre
E	Modulus of elasticity
r	Non-dimensional distance of neutral axis from extreme compression fibre $=H_1/H$
m	Parameter
M	Bending moment at the section
M_{yd}	Dynamic yield moment for I-section $=\frac{\sigma_{yd}}{6H}\left[BH^3-(B-b)(H-2h)^3\right]$
M_{1yd}	Dynamic yield moment for rectangular section $=\sigma_{yd}BH^2/6$
$\overline{M}$	M/M_{yd} = shape factor of I-section when yield stress is σ_{yd}
P	Axial force on the section
P_{yd}	Yield force for I-section $=\sigma_{yd}[BH-(B-b)(H-2h)]$
P_{1yd}	Yield force for rectangular section $=\sigma_{yd}BH$
$\overline{P}$	P/P_{yd}
R	Radius of curvature

Greek symbols

α,β,γ	Parameters
$\alpha_1,\beta_1,\gamma_1$	Parameters
ε	Normal strain
ε_y	Yield strain
$\varepsilon_m=k\varepsilon_y$	Strain in extreme fibre
$\varepsilon_h=k_1\varepsilon_y$	Strain corresponding to end of yielding and beginning of strain-hardening
$\varepsilon_u=k_2\varepsilon_y$	Ultimate strain
σ	Normal stress
σ_{yd}	Dynamic yield stress
σ_{ud}	Dynamic ultimate stress

Subscripts

b	Bottom
d	Dynamic
h	Beginning of hardening
t	Top
u	Ultimate
y	Yield

are three zones in the idealised diagram: elastic zone from $k=0$ to $k=1$; yield zone without any strain-hardening from $k=1$ to $k=k_1$; and the strain-hardening zone from $k=k_1$ to $k=k_2$, where, $k\varepsilon_y$ is the strain. The stress in the strain-hardening range, σ_d, at any strain, $\varepsilon=k\varepsilon_y$ $(k_1\le k\le k_2)$, can be obtained from the following relations:

$$(\sigma_d-\sigma_{yd})=(\sigma_{ud}-\sigma_{yd})m \quad \text{for} \quad \text{Linear-Hardening} \tag{1}$$

$$(\sigma_d-\sigma_{yd})=(\sigma_{ud}-\sigma_{yd})m(2-m) \quad \text{for} \quad \text{Parabolic-Hardening} \tag{2}$$

where,

$$m=\left(\frac{\varepsilon-\varepsilon_h}{\varepsilon_u-\varepsilon_h}\right)=\left(\frac{k-k_1}{k_2-k_1}\right) \tag{3}$$

The parameter k is the ratio of strain to yield strain. The suffix d in the above expressions has been used to indicate dynamic values. The stress–strain curve can be used for high strength steel by substituting $k_1=1$ and many other materials can be easily represented by these equations for different values of the parameters. The strain-softening portion of the curve has been ignored in the present analysis.

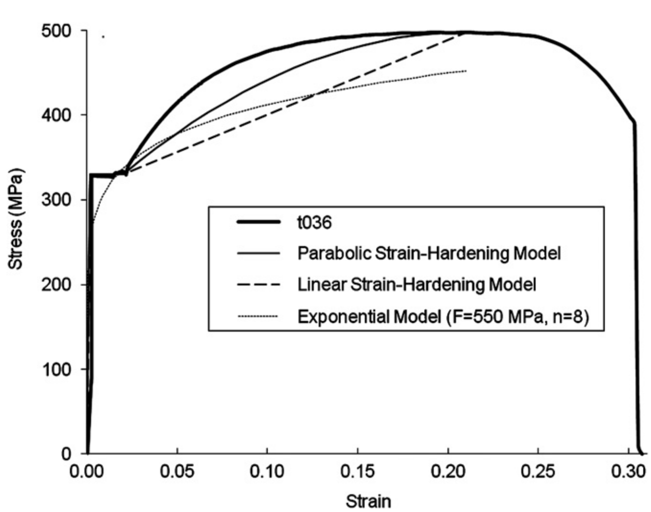

Fig. 1. Experimental stress–strain curve [19] and different models of mild steel.

3. Bending moment–axial force (*M–P*) interaction

A symmetrical I-section of beam of width of flange, B, thickness of flange, h, overall depth, H, and the thickness of web, b, has been considered for studying the interaction of a bending moment, M, and an axial force, P (Fig. 2). The geometry of the section is defined by the following non-dimensional parameters:

$$\left.\begin{aligned} \alpha &= h/H \\ \beta &= (B-b)/B \\ \gamma &= b/B \\ \gamma_1 &= \alpha\beta(1-\alpha) \end{aligned}\right\} \tag{4}$$

The range of the above non-dimensional parameters for wide I-sections of AISC is as follows:

$0.019\le\alpha\le0.219$; $0.828\le\beta\le0.970$; $0.030\le\gamma\le0.172$ and $0.018\le\gamma_1\le0.142$.

The I-section converts to the rectangular section when, $\alpha=0$, $\beta=0$, $\gamma=1$, $\gamma_1=0$.

The bending moment is assumed to cause tension at the bottom face. The axial force considered in the present analysis is tensile and the same relations can be used for a compressive axial force because

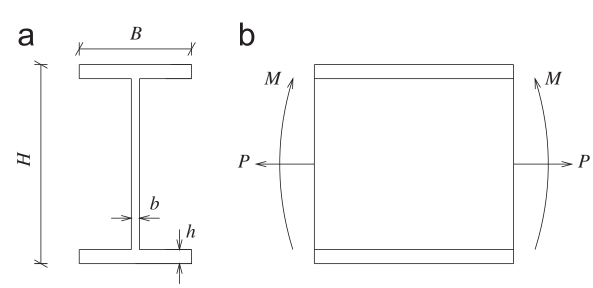

Fig. 2. (a): Section of beam, (b) an element subjected to external pull and bending moment.