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Multicriteria optimization of cold-formed thin-walled beams with generalized open shape under different loads

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ABSTRACT

The paper presents an analysis of the optimal design of cold-formed beams with generalized open shapes under pure bending, uniformly distributed loads, concentrated loads and axial loads with constant bending moment. The optimization problem includes the cross section area as the first objective function and the deflection of a beam as the second one. The geometric parameters of cross sections are selected as design variables. The set of constraints includes global stability condition, selected forms of local stability conditions, strength condition and technological and constructional requirements in a form of geometric relations. The strength and stability conditions are formulated and analytically solved using mathematical equations. The optimization problem is formulated and solved with help of the Pareto concept of optimality. The numerical procedure, based on the Messac normalized constraint method, include discrete, continuous and discrete-continuous sets of design variables. Results of the numerical analysis for different loads of beams with monosymmetrical cross section shapes are presented in tables.

1. Introduction

The thin-walled beams are distinguished by their good strength properties, relatively low weight and the ability to carry heavy loads. The main advantage of these structures is the beneficial relationship between the weight and loads carrying capabilities. The cold-formed thin-walled beams are capable of meeting rigorous requirements imposed by users in different branches of mechanical industry and civil engineering. The biggest value of these structures is their efficiency, as large increment of strength may be obtained through by appropriate choice of cross section shape, with minimal or no weight increase. The thin-walled beams can be therefore considered as an area with great research potential, where innovation can lead to very practical improvements. There are many significant works about cold-formed thin-walled structures, including works of Davies [1], Ghersi et al. [2], Hancock [3] and many others papers and monographs.

From theoretical point of view, the thin-walled beam structures have also some limitations that must be address during a design process. The most important is susceptibility of these structures to global and different forms of local buckling. In most practical cases, calculations of critical loads require applying complex methods of finding satisfactory solutions for the stability state of the thin-walled beams. These calculations may be done analytically or numerically, what was presented by Mohri [4].

The advantages of thin-walled beams can be better utilized, and their faults can be minimized, if their basic geometric parameters are calculated with the help of structural optimization. Most of the works in the area of optimal design are focused on so-called single criterion scalar optimization, where the most commonly used optimality criterion is weight of a structure. Such a criterion is usually connected with economic aspects as material, manufacturing and application costs to a certain degree depend on weight.

Single criterion scalar optimization has limited practical value, as in most engineering cases, several noncomparable criteria must be considered in order to reach the optimal design of a structure. These requirements lead to the need for multicriteria optimization, where the structure is analyzed in the context of several, often conflicting, criteria. In the result, such an optimal design is closer to technical reality and it much better describes real conditions of structures and their behavior.

Procedures of using bicriteria optimization for the optimal design of thin-walled cold-formed beams with open different cross sections were adapted by Kasperska et al. [5–7], Magnucki and Ostwald [8,9], Ostwald and Magnucki [10], Ostwald et al. [11], Rodak and Ostwald [12], Ostwald and Rodak [13–15]. The presented paper is continuation and summary of the previous works. Problems of stability and optimization of open cold-formed beams

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with special cross sections are presented by Lewiński and Magnucki [16] and Magnucki et al. [17].

2. Models of thin-walled beams with open cross sections

The current research approaches to the problem of optimal design of thin-walled beams take into account three different cases of beam loads:

– pure bending (two equal moments M [kN m] applied to the ends of a beam),
– uniformly distributed load q [kN/m],
– concentrated load P [kN].

These loads are illustrated in Fig. 1.

In the third case of beam load, a special vertical rib is applied in the point where the concentrated load P is weighed. This rib prevents the local buckling of a beam.

The generalized cross section is characterized by parameters a, b, c, d, t and additionally angle α describing relationships between the lip and the flange. For the generalized monosymetrical cross section presented in Fig. 2 the total area A and the geometric stiffness for Saint-Venant torsion I_s are in the form:

$$A = 2t\sum_{i=1}^{6} l_i = 2t(a+b+c+d), \quad I_s = \frac{2}{3}t^3\sum_{i=1}^{6} l_i = \frac{2}{3}t^3(a+b+c+d),$$

where l_i (i=1, …, 6) has the following lengths: $l_1=a-t$, $l_2=d$, $l_3=t$, $l_4=b$, $l_5=t$, $l_6=c-t$.

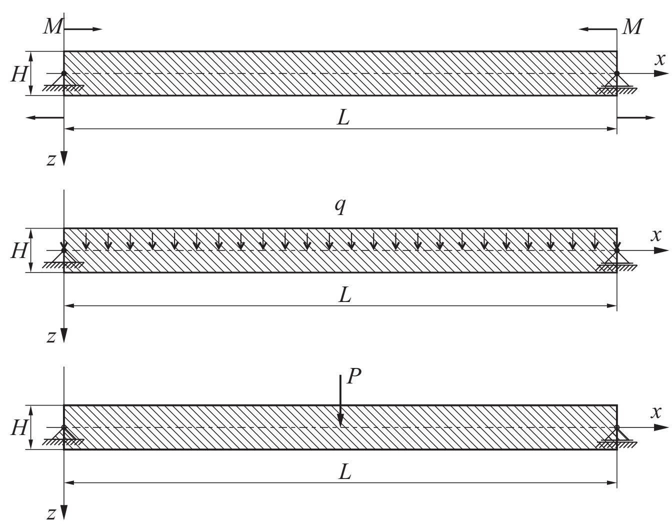

Fig. 1. Models of the thin-walled beams.

The auxiliary coordinates of point $B(y_B, z_B)$ are the following:

$$y_B = \frac{d^2+2td-b(b-2d)-2t(b-d)-(c-t)[2(b-d)-(c-t)\cos\alpha]}{2(a+b+c+d)}, \quad z_B = 0.$$

The moments of inertia I_y and I_z of the cross section area in respect to the y and z axes are as follows:

$$I_y = 2t\sum_{i=1}^{6}\frac{1}{3}l_i(z_{i-1}^2+z_{i-1}z_i+z_i^2), \quad I_z = 2t\sum_{i=1}^{6}\frac{1}{3}l_i(y_{i-1}^2+y_{i-1}y_i+y_i^2),$$

where y_i and z_i mean the coordinates of the specific cross section points (points from 0 to 6, see Fig. 2):

$$y_0 = y_B,\ y_1 = y_B,\ y_2 = -d+y_B,\ y_3 = -d+y_B,\ y_4 = b-d+y_B,$$
$$y_5 = b-d+y_B,\ y_6 = b-d-(c-t)\cos\alpha+y_B, z_0 = 0,$$
$$z_1 = -(a-t),\ z_2 = -(a-t),\ z_3 = -a,\ z_4 = -a,$$
$$z_5 = -(a-t),\ z_6 = -[a-t-(c-t)\sin\alpha].$$

The main pole S coordinates are:

$$y_S = y_B + \frac{I_{y\omega_B}}{I_y},$$

where

$$I_{y\omega_B} = 2t\sum_{i=1}^{6}\frac{1}{3}l_i\left(z_{i-1}\omega_{B,i-1}+\frac{1}{2}z_{i-1}\omega_{B,i}+\frac{1}{2}z_i\omega_{B,i-1}+z_i\omega_{B,i}\right)$$

and ω_{Bi} means auxiliary sectorial coordinates with the pole B:

$$\omega_{B,0} = 0,\ \omega_{B,1} = 0,\ \omega_{B,2} = -(a-t)d,\ \omega_{B,3} = \omega_{B,2}+td,$$
$$\omega_{B,4} = \omega_{B,3}+ba,\ \omega_{B,5} = \omega_{B,4}+(b-d)t,$$
$$\omega_{B,6} = \omega_{B,5}+(c-t)[(b-d)\sin\alpha-(a-t)\cos\alpha].$$

The sectorial moment of inertia of the generalized cross section has the form:

$$I_\omega = 2t\sum_{i=1}^{6}\frac{1}{3}l_i(\omega_{i-1}^2+\omega_{i-1}\omega_i+\omega_i^2),$$

where ω_i means the sectorial coordinates with the pole S: $\omega_i=\omega_{B,i}-(y_S-y_B)z_i$ (for points i=0 to 6).

For the generalized antisymmetrical cross section (Fig. 2) the following additional formulas are applied. The product of inertia in respect to the y and z axes is as follows:

$$I_{yz} = \frac{2}{3}t\sum_{i=1}^{6} l_i\left(y_{i-1}z_{i-1}+\frac{1}{2}y_{i-1}z_i+\frac{1}{2}y_iz_{i-1}+y_iz_i\right),$$

where the coordinates of the specific cross section points z_i are the same as in the monosymmetric cross section, whereas coordinates of y_i are:

$$y_0 = y_1 = 0,\ y_2 = -d,\ y_3 = -d,\ y_4 = b-d,$$
$$y_5 = b-d,\ y_6 = b-d-(c-t)\cos\alpha.$$

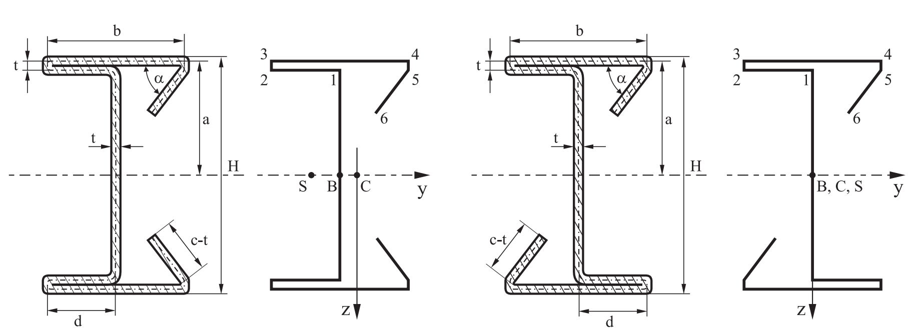

Fig. 2. Generalized monosymmetrical and antisymmetrical cross section.