Thin-Walled Structures 58 (2012) 40–50

Contents lists available at SciVerse ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Elastic buckling of elliptical tubes subjected to generalised linearly varying stress distributions

J.M. Abela, L. Gardner*

Department of Civil and Environmental Engineering, Imperial College London, London SW7 2AZ, United Kingdom

ARTICLE INFO

Article history:
Received 2 March 2012
Accepted 24 April 2012
Available online 18 May 2012

Keywords:
Biaxial bending
Combined bending and axial compression
Cross-sectional classification
Elastic buckling
Elliptical hollow sections
Instability
Oval hollow sections

ABSTRACT

The structural behaviour of elliptical hollow sections has been examined in previous studies under several loading conditions, including pure compression, pure bending and combined uniaxial bending and compression. This paper examines the elastic buckling response of elliptical hollow sections under any linearly varying in-plane loading conditions, including the most general case of combined compression and biaxial bending. An analytical method to predict the elastic buckling stress has been derived and validated against finite element results. The predictive model first identifies the location of the initiation of local buckling based on the applied stress distribution and the section geometry. The critical radius of curvature corresponding to this point is then introduced into the classical formula for predicting the elastic local buckling stress of a circular shell. The obtained analytical results are compared with results generated by means of finite element analysis. The comparisons between the analytical and numerical predictions of elastic buckling stress reveal disparities of less than 2.5% for thin shells and, following an approximate allowance for the influence of shear, less than 7.5% for thick shells.

1. Introduction

Research activity in the area of elliptical sections has increased in recent years due to their emergence as hot-rolled structural products. Elliptical hollow sections (EHS) combine the elegance of circular hollow sections (CHS) with the improved structural efficiency in bending associated with sections of differing flexural rigidities about the two principal axes. This behaviour has been exploited in a number of recent projects that have adopted EHS as structural elements, such as the Honda Central Sculpture in Goodwood, UK, the Society Bridge in Braemar, UK [1] and the airport at Barajas in Madrid, Spain [2]. EHS were also included in the latest edition of the European product standard EN 10210 [3] and in the BCSA/SCI Eurocode member resistance tables [4]. A review of research into the structural behaviour of elliptical hollow sections, together with a description of recent practical applications, may be found in [5]. Further subsequent work on the buckling response of EHS members has also been performed [6–12].

2. Literature review

The focus of the present study is the elastic buckling response of elliptical tubes under linearly varying in-plane stress distributions, including the most general case of combined axial compression plus biaxial bending. First, previous investigations of the elastic buckling response of EHS under isolated loading cases, as well as compression plus uniaxial bending, are reviewed.

2.1. Compression

Marguerre [13] made the first attempt at representing the buckling behaviour of cylindrical shells of variable curvature under compression. The work was later continued by Kempner [14] and Hutchinson [15]. Kempner's work [14] concluded that the elastic buckling stress of an oval hollow section (OHS) could be accurately predicted as the buckling stress of a circular hollow section (CHS) with a radius equal to the maximum radius of curvature of the OHS. This solution was shown to be a lower bound. Hutchinson [15] found that this approach could also be applied to elliptical hollow sections (EHS), provided that the shell is sufficiently thin. The proposals were later confirmed by experiments carried out by Tennyson et al. [16].

Further investigations have been carried out by Zhu and Wilkinson [17], Chan and Gardner [18], Ruiz-Terán and Gardner [19] and Silvestre [20]. These studies confirmed that Kempner's approach [14] of basing the elastic buckling stress of an EHS on that of a CHS with a radius equal to the maximum radius of curvature of the ellipse is acceptable but with increasing errors for higher aspect ratios and thicker sections. Analyses of the

* Corresponding author. Tel.: +44 207 594 6031.
E-mail address: leroy.gardner@imperial.ac.uk (L. Gardner).

0263-8231/$ - see front matter
http://dx.doi.org/10.1016/j.tws.2012.04.016

elastic buckling response of EHS using generalised beam theory (GBT) was undertaken by Silvestre [20]. Both Ruiz-Terán and Gardner [19] and, using GBT, Silvestre [20] proposed modifications to the Kempner equation to achieve more accurate predictions of the elastic buckling stress of EHS of various aspect ratios and shell thicknesses.

2.2. Bending

Investigations of elliptical cylinders subjected to major axis bending were carried out by Heck [21] and Gerard and Becker [22] where it was observed that although the maximum compressive stress occurs at the stiffest part of the cross-section (which is most resistant to buckling), the critical radius of curvature occurs at a point between the maximum and minimum radii of curvature. Gerard and Becker [22] derived the critical radius for major axis bending to be $r_{\mathrm{cr},b,\mathrm{ma}}=0.649a^2/b$, where a and b are the half major and half minor axis dimensions of an EHS, respectively, by optimising the function composed of the varying curvature expression and the elastic bending stress distribution. For minor axis bending, the location of initiation of buckling was found to be at the same location as for an EHS under pure compression (i.e., $r_{\mathrm{cr},b,\mathrm{mi}}=a^2/b$). Once the critical radius of curvature (i.e., the location of the initiation of buckling) has been determined, the elastic buckling stress of the EHS can again be calculated by means of the elastic buckling expression for a CHS.

2.3. Combined actions

The performance of EHS under combined compression and uniaxial bending was investigated by Gardner et al. [23], following studies of the pure compression and pure bending cases previously presented [18,24]. The critical radius of curvature for an EHS under compression and minor axis bending was found to be similar to the pure compression and pure minor axis bending cases, where $r_{\mathrm{cr,mi}}$ is expressed as a^2/b. Under combined compression and major axis bending, the critical radius will shift towards the centroidal axis as the compressive part of the loading increases, and an expression in terms of ψ, the ratio of the end stresses, was proposed. A simplification of this expression for $a/b=2$, considering a conservative linear transition between the pure compressive and pure major axis bending critical radii, is:

$$r_{\mathrm{cr,ma}}=r_{\mathrm{cr},b,ma}+(r_{\mathrm{cr},c}-r_{\mathrm{cr},b,ma})\left(\frac{\psi+1}{2}\right) \quad (1)$$

where, $r_{\mathrm{cr},b,\mathrm{ma}}$ and $r_{\mathrm{cr,c}}$ are the critical radii of curvature for pure major axis bending and pure compression, respectively, and ψ is the ratio of end stresses and is in the range of $-1\le\psi\le 1$.

3. Analytical study of elastic buckling of EHS under combined actions

The elastic buckling stress σ_{cr} of an EHS may be found from the classical buckling stress [14,22] expression:

$$\sigma_{cr}=\frac{Et}{r_{cr}\sqrt{3(1-\nu^2)}} \quad (2)$$

where, r_{cr} corresponds to the point of initiation of local buckling in the cross-section, which depends on the applied stress distribution, E is the Young's modulus, t is the shell thickness and ν is the Poisson's ratio.

The present section will focus on finding the location of the critical radius of curvature when an EHS is subjected to any generalised linearly varying in-plane stress distribution. The most general form of such loading on a cross-section is the case of

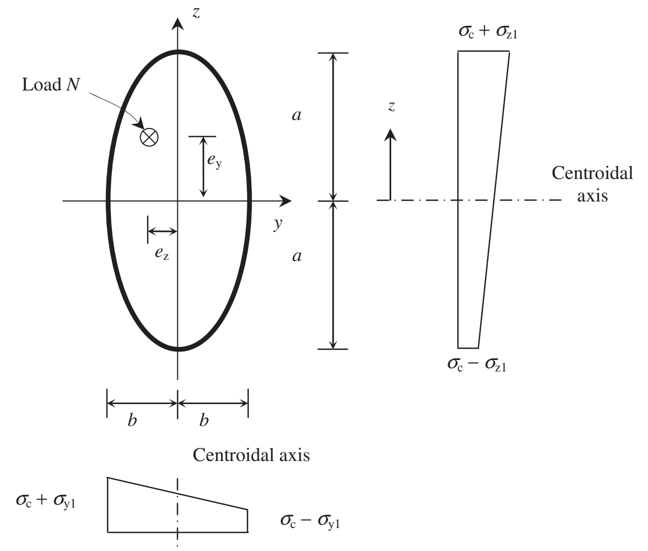

Fig. 1. EHS under combined compression and biaxial bending and stress distributions along the centroidal axes.

combined compression and biaxial bending. An analytical model that yields the exact location of the initiation of local buckling in an elliptical cross-section is presented.

Combined compression and bending can be achieved on a cross-section by applying a compressive force at eccentricities (e_z and e_y) to the centroid of the section, producing the stress distributions illustrated in Fig. 1. The location of the initiation of local buckling and the corresponding critical radius r_{cr} for this load combination may be found by optimising the product of the stress function σr, which is the product of the elastic bending stress distribution and the radius of curvature; i.e., the location at which the stress causing local buckling is minimum is being sought. This method to find the critical radius was originally used by Gerard and Becker [22] for the case of pure major axis bending, as described in Section 2.2.

It is therefore necessary to define equations for stress and the radius of curvature in order to assemble the stress function. An EHS subjected to a combination of a compressive load and moments about both axes would have a total elastic stress distribution defined by:

$$\sigma=\sigma_c+\sigma_{z1}\frac{z}{a}+\sigma_{y1}\frac{y}{b} \quad (3)$$

where $\sigma_c=N/A$ is the uniform compressive stress, in which N is the applied compressive load and A is the cross-sectional area, $\sigma_{z1}(z/a)$ is the linearly varying stress associated with the major axis, $\sigma_{y1}(y/b)$ is the linearly varying stress associated with the minor axis, $\sigma_{z1}=Ne_ya/I_y$ at $z=a$, where I_y is the second moment of area about the major axis and $\sigma_{y1}=Ne_zb/I_z$ at $y=b$, where I_z is the second moment of area about the minor axis; see also Fig. 1.

The general expression for the radius of curvature of an EHS is:

$$r=\frac{a^2}{b}\left\{1-\left[1-\left(\frac{b}{a}\right)^2\right]\left(\frac{z}{a}\right)^2\right\}^{\frac{3}{2}}. \quad (4)$$

The mathematical optimisation of the stress function σr:

$$\sigma r=\left(\frac{N}{A}+\sigma_{z1}\frac{z}{a}+\sigma_{y1}\frac{y}{b}\right)\frac{a^2}{b}\left\{1-\left[1-\left(\frac{b}{a}\right)^2\right]\left(\frac{z}{a}\right)^2\right\}^{3/2} \quad (5)$$