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Impact loading of ductile rectangular plates

Norman Jones

Impact Research Centre, Department of Engineering, University of Liverpool, Liverpool, UK

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ABSTRACT

In many industries, rigid-plastic methods of analysis are a useful design aid for safety calculations, hazard assessments, security studies and forensic investigations of ductile structures, which are subjected to large dynamic loads producing an inelastic response. This paper examines the behaviour of a rectangular plate struck at the centre by a rigid mass impact loading. A theoretical method has been developed previously for arbitrarily shaped plates which retains the influence of finite transverse displacements, or geometry changes. It is used in this paper to predict the maximum permanent transverse displacements and response duration of plates having boundary conditions characterised by a resisting moment mM_0 around the entire boundary, where $m=0$ and 1 give the two extreme cases of simply supported and fully clamped supports, respectively.

The theoretical predictions are compared with some experimental data recorded on fully clamped metal rectangular plates having a range of aspect ratios and struck by masses travelling with low impact velocities up to nearly 7 m/s and which produce large ductile deformations without failure. The theoretical analysis gives reasonable agreement with the corresponding experimental data for masses having blunt, conical and hemispherical impact faces.

For sufficiently large initial impact energies, the projectile would perforate a plate and, for completeness, a useful design equation is presented which predicts perforation energies larger than all of the test data, as expected.

1. Introduction

Simple and reliable theoretical methods are still valuable for design purposes, particularly for preliminary design and hazard assessments, and for forensic investigations after accidents. A theoretical rigid-plastic method was developed in [1], which retained the influence of large transverse displacements (i.e., geometry changes, or membrane effects) and which has been used to predict the maximum permanent transverse displacements, or damage, of ductile beams, circular and rectangular plates when subjected to a pressure pulse causing plastic strains. It was shown how this method can be simplified with an approximate yield condition to predict useful design equations, which circumscribe and inscribe the predictions of an exact yield criterion. This method was also used to examine the impulsive, or blast, loading of rectangular plates, and good agreement was found with experimental results recorded on ductile metal plates having various aspect ratios [2–4]. The method was extended to obtain the response of circular plates [4–6] and square plates [6] when struck by a solid mass at the centre, and again good agreement was reported with the maximum permanent transverse displacements observed in experimental tests on ductile metal plates.

This paper extends the above theoretical method to obtain the maximum permanent transverse displacements, or ductile damage, for a rectangular plate struck by a rigid mass at the centre. It turns out that a relatively simple equation was obtained which gives good agreement with experimental data recorded on ductile metal rectangular plates having a range of aspect ratios from 0.4 to 1 and reported in [7]. Thus, design equations are now available for predicting the maximum permanent transverse displacements, or damage, for circular plates and rectangular plates (including square plates and beams) subjected to pressure pulses (including the limiting case of an impulsive loading) or central solid mass impacts. Moreover, these equations have been tested against experiments on ductile metal plates and are therefore suitable for design purposes, safety calculations, security studies and hazard assessments.

The next section of this paper outlines the theoretical method which is used in Section 3 to examine the behaviour of a ductile rectangular plate subjected to a mass impact loading at the mid-span. Section 4 discusses briefly the experimental details of the data obtained on mild steel rectangular plates struck by a mass at the plate centre which produces large ductile deformations without any failure. Sections 5 and 6 contain a discussion and conclusions, respectively.

2. Theoretical method for dynamic loading of plates

A theoretical procedure was developed in [1], to study the response and predict the permanently deformed profile of an

E-mail addresses: njones@ijie.fsnet.co.uk, norman.jones@liv.ac.uk

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Nomenclature

a	defined in the Appendix
d	diameter of projectile
m	dimensionless moment resistance at supports, $m=0$ and 1 for simply and fully clamped supports, respectively
q	Cowper Symonds exponent (Eq. (14))
t	time
w	transverse displacement
x, y	Cartesian coordinates (Fig. 2)
A	surface area of a plate
$2B$	width of a rectangular plate (Fig. 2)
D	Cowper Symonds coefficient (Eq. (14))
E_r	energy ratio (Eq. (13))
G	mass of a projectile or striker
H	plate thickness
K	dimensionless initial kinetic energy (Eq. (15))
$2L$	length of a rectangular plate (Fig. 2)
M_0	plastic collapse moment per unit length ($\sigma_0 H^2/4$)
M_r, M_θ	radial and circumferential bending moments per unit length
N_0	plastic collapse force per unit length ($\sigma_0 H$)
N_r, N_θ	radial and circumferential membrane forces per unit length
R	radius of circular plate
S	span
T	response time
V	volume of material
V_0	initial impact or impulsive velocity
W	transverse displacement at centre of rectangular and circular plates (Fig. 2)
W_f	maximum permanent transverse displacement
α	defined by Eq. (5a)
β	aspect ratio (Eq. (5b))
γ, γ_c	mass ratios (Eqs. (5c) and (17) for rectangular and circular plates, respectively)
ε_f	engineering rupture strain in tension
$\dot{\varepsilon}$	strain rate (Eq. (14))
$\overline{\eta}$	pressure pulse ratio (Appendix)
κ_r,κ_θ	radial and circumferential changes of curvature
λ	$\rho V_0^2 L^2/M_0$ for a rectangular or square plate, Eq. (A.4) for a circular plate
μ	mass per unit surface area of a plate
ρ	density of plate material
σ_0,σ_0'	static and dynamic flow stresses
σ_u	static ultimate tensile stress
Ω	dimensionless initial kinetic energy (Eq. (12))

arbitrarily shaped ductile plate, when subjected to large static or dynamic loads which produce plastic strains. The material is assumed to be rigid, perfectly plastic with a yield stress σ_0 and the plate has a uniform thickness H. The governing equations can be simplified for an impact loading and written in the form

$$-G\ddot{W}\dot{W}-\int_A \mu\ddot{w}\dot{w}\,dA=\int_A\{(M_r+wN_r)\dot{\kappa}_r+(M_\theta+wN_\theta)\dot{\kappa}_\theta\}\,dA + \sum_{m=1}^{n}\int_{C_m}(M_r+wN_r)(\partial\dot{w}/\partial r)_m\,dC_m + \sum_{u=1}^{v}\int_{C_u}Q_r(\dot{w})_u\,dC_u \quad (1)$$

where G is an impact mass, and μ is the mass per unit surface area of a plate. The transverse displacement of a plate is w, while $\dot{w}$ and $\ddot{w}$ are the associated velocity and acceleration. W is the transverse displacement at the plate centre which is immediately underneath a striking mass.

The terms on the left hand side of Eq. (1) are the work rate due to the inertia forces, where A is the surface area of a plate. The first term on the right hand side of Eq. (1) is the energy dissipated in any continuous deformation fields. The second term gives the energy dissipated in n plastic bending hinges, each having an angular velocity $(\partial\dot{w}/\partial r)_m$ across a hinge of length C_m. The final term is the plastic energy absorption in v transverse shear hinges, each having a velocity discontinuity $(\dot{w})_u$ and a length C_u. Eq. (1) ensures that the external work rate equals the internal energy dissipation.

The general method has been used to study the dynamic plastic response of beams, and of circular, square and rectangular plates subjected to dynamic pressure pulses and also blast loadings [1–4], and for beams, circular and square plates struck at the mid-span by a rigid mass [4–6]. It is used in this paper to examine a rectangular plate struck by a rigid mass at the centre, and, since large ductile deformations are studied without any material failure, or perforation, the last (transverse shear) term in Eq. (1) is not considered further. Thus, the yield condition consists of four generalised stresses (M_r,M_θ,N_r,N_θ), which can be related by the limited interaction surface shown in Fig. 2 of [6]. However, if a deformation profile consists only of rigid regions separated by plastic hinges, then the exact yield condition in Fig. 1 governs plastic flow at the hinge lines. A square yield condition circumscribes the exact yield condition (maximum normal stress yield criterion), while another one which is 0.618 times as large would

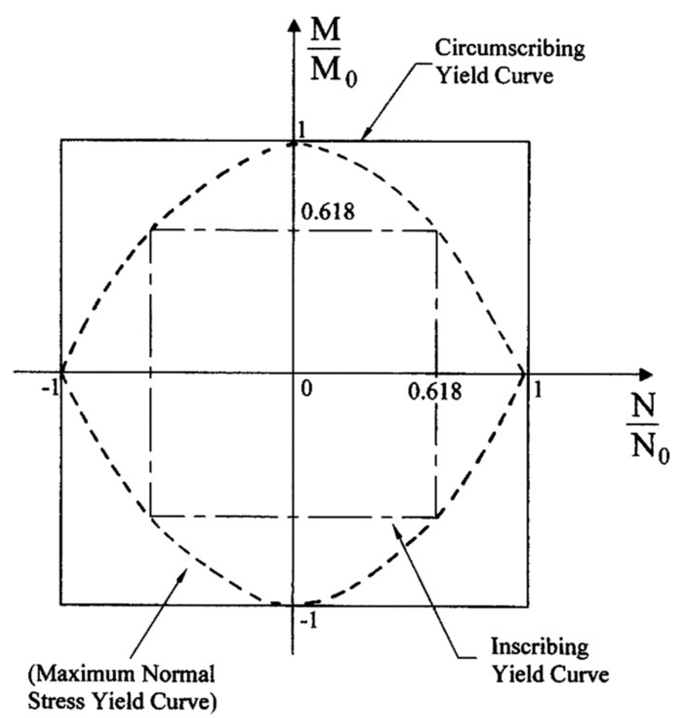

Fig. 1. Yield conditions at the plastic hinge lines (including the supports for $m \neq 0$) which develop within the rectangular plate in Fig. 2.