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An inelastic finite strip method for thin-walled compression members

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ABSTRACT

Over the years the finite strip method (FSM) has proved to be an invaluable tool in the study of buckling modes in thin-walled structural members. This paper presents a formulation of the FSM, which is able to predict the buckling stresses of initially perfectly straight thin-walled inelastic members under uniform compression. Plasticity is accounted for by means of plastic flow equations. Previous shortcomings of plastic flow theory with respect to the modeling of buckling problems are overcome by deriving an expression for the inelastic shear stiffness from second order considerations.

1. Introduction

Early work in the development of the finite strip method (FSM) was carried out by Wittrick [33], and Williams and Wittrick [32]. However, the FSM in its current form, often referred to as the semi-analytical formulation, is attributed to Przemieniecki [29], Planck and Wittrick [28], and Cheung [11].

Graves Smith and Sridharan [18] and Hancock [19] extended the method to study the elastic post-buckling behavior of thin plates. Fan [14] and later Lau and Hancock [22] developed a version of the FSM using spline displacement functions, while Dawe [12] used curved finite strips.

Over the past decades the FSM in its various forms has been used extensively by many researchers and is credited with greatly contributing to our understanding of buckling modes in thin-walled members.

The premise of the FSM in its basic form is that the buckling deformations in the direction of the longitudinal axis of the member can be represented by a sinusoidal function. Adopting this as a given, the member is then subdivided in the transverse direction, giving rise to a number of strips separated by nodal lines (Fig. 1). Cubic polynomials are used to represent the out-of-plane displacements along a transverse line. This method holds an advantage in terms of computational efficiency over the finite element method, which requires a much higher number of elements as a result of the necessary discretization of the geometry in both the longitudinal and transverse directions. Although different boundary conditions have been studied by proposing displacement functions representing different longitudinal shapes [9], the FSM was originally developed for strips which are simply supported at the ends, and the paper will stay within this scope.

The FSM is ideally suited for implementation into computer software: CUFSM [30], developed at John Hopkins University, and ThinWall [27], developed at the University of Sydney, both implement the FSM for elastic materials. Fig. 2 shows a typical output diagram for ThinWall. The section under consideration is a lipped channel and a single sine half-wave is assumed for the longitudinal displacements. Fig. 2, which plots the elastic buckling stress versus the buckling half-wavelength, shows three distinct branches in the output curve. The buckling mode associated with the shorter wavelengths is the local mode and the minimum in the curve indicates the local buckling stress and the local buckling half-wavelength of a locally buckled 'cell' (Fig. 3). The solution for the intermediate wavelengths corresponds with distortional buckling, while the longer wavelengths are associated with overall (column) buckling.

The inelastic finite strip method presented in this paper was implemented in MATLAB and required a mere 170 lines of code, thus providing a valid, economical alternative in cases where commercial finite element packages are unavailable or deemed too expensive. Furthermore, finite element packages usually only provide the option of performing an *elastic* buckling analysis, which is a perturbation analysis based on the initial (elastic) material properties. Few, if any, commercial packages allow for the user to conduct an inelastic buckling analysis, accounting for loss of stiffness at higher stress levels. As an added advantage, a modified version of the finite strip method, the 'constrained finite strip method' allows for a modal decomposition of the results, which is difficult to obtain through finite elements [1]. This modal decomposition is an important tool to complement the Direct Strength Method for thin-walled structural members [25].

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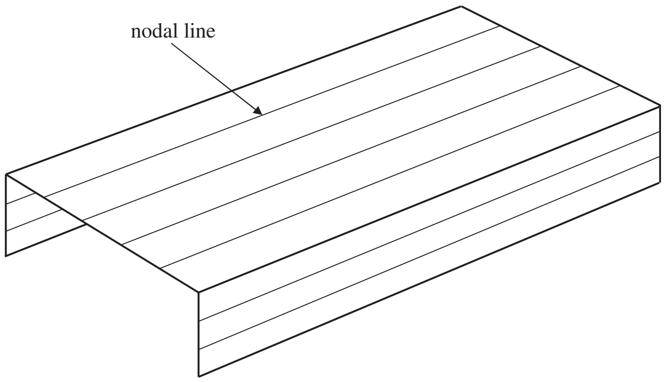

Fig. 1. Finite strips.

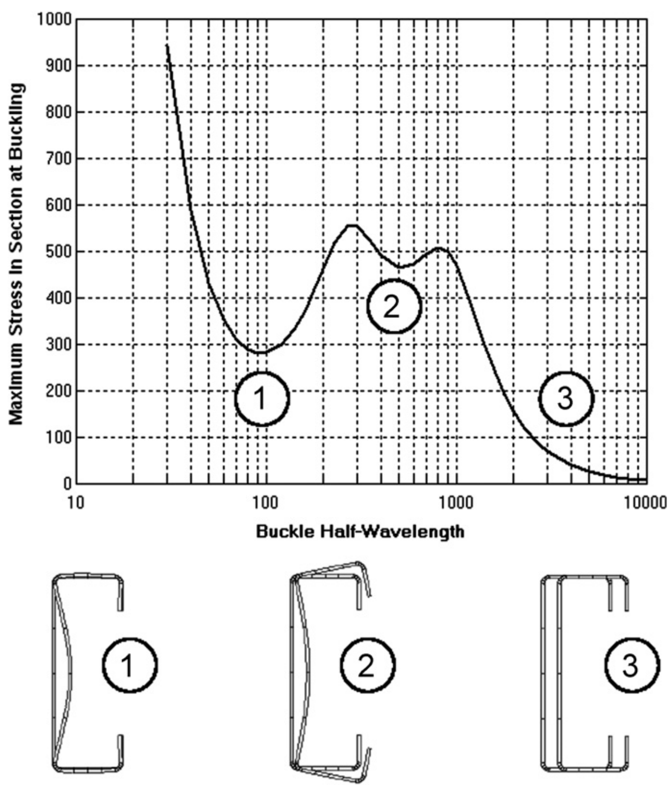

Fig. 2. Typical FSM results.

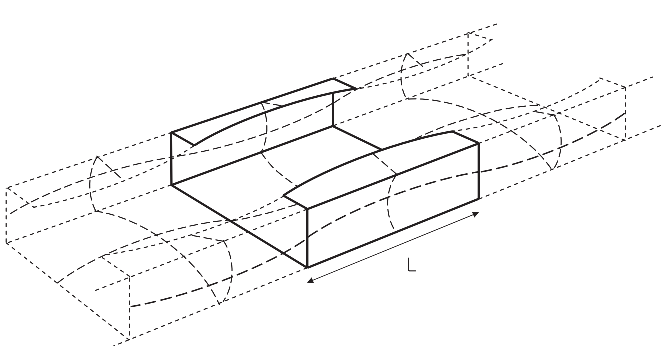

Fig. 3. Locally buckled shape.

Many structural, aeronautical, nautical, and mechanical applications involve thin-walled members of non-linear metals, such as aluminum or stainless steel. Gradual yielding in the stress–strain curve, associated with a gradual loss of stiffness affects the buckling behavior of these members. Lau and Hancock [23] developed two versions of the FSM, which account for elastic–plastic behavior. A first version is based on plastic deformation theory [8], assuming a fixed relationship between stresses and total strains. Although deformation theory is generally considered flawed in its concept and inferior to plastic flow theory, Lau and Hancock reported reasonable agreement with the experiment. Secondly, the researchers developed an FSM formulation based on the plastic flow equations of Handelman and Prager [20] and found that the model considerably overestimated the buckling stresses of plates, a finding which echoed the results of Handelman and Prager's [20] plate theory. The short-comings of plastic flow theory in its ability to model buckling problems are well-documented and are usually referred to as the 'plastic buckling paradox'. The essence of this problem lingers in the fact that plastic flow theory wrongfully dictates a totally elastic relationship between shear stresses and shear strains at the onset of buckling [21]. Due to this elastic 'locking-in' of the shear stresses with the shear strains, the shear stiffness and consequently the buckling stresses are grossly overestimated.

Bradford and Azhari [10] developed their own inelastic finite strip formulation, which included the use of bubble functions: extra modes associated with nodeless degrees of freedom. They demonstrated that the addition of these bubble functions has a beneficial effect on the convergence of the solution, similar to using a larger amount of strips. The constitutive relations adopted in their model are based on the deformation theory of plasticity and borrow from the plate theories of Bijlaard [8] and Stowell [31]. In a later paper Azhari et al. [2] extended these concepts to include thickness-tapered plates.

It is noted that alternatives to the finite strip method are available and have been used successfully to study the stability of thin-walled inelastic compression members. In this respect the finite element method using shell elements provides a particularly versatile and popular option. Recent work using shell finite elements in this area includes: Gardner and Nethercot [15], Lecce and Rasmussen [24], Mahmud et al. [26], Becque and Rasmussen [5,6], and Zhu and Young [34].

Gonçalves and Camotim [16] and Gonçalves et al. [17] on the other hand chose to employ generalized beam theory for the inelastic bifurcation analysis of uniformly compressed thin-walled members. The authors developed two formulations, with one being based on plastic deformation theory, while the other incorporated plastic flow theory. Examples indicated that the flow based theory consistently yielded much higher predictions than the deformation based theory.

This paper aims to present an inelastic version of the FSM, which is unaffected by the plastic buckling paradox. To overcome the paradox, a relationship is first derived between shear stresses and shear strains from second order considerations i.e. by considering an infinitesimal plate element in its shear-deformed state in the presence of a compressive stress. In a subsequent step, an FSM for inelastic thin-walled columns is formulated with the use of this information.

2. Inelastic shear stiffness

The principles used in the derivation of the inelastic shear stiffness were first proposed in Becque [4] and later amended in Becque [3] and the reader is referred to these publications for a more detailed description of the fundamentals.

Fig. 4 depicts a plate element under uniform axial compression before local buckling. The plate consists of an inelastic material and consequently, the total axial strain increment $\dot{\varepsilon}_x$ under an