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Global buckling of thin-walled simply supported columns: Analytical solutions based on shell model

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ABSTRACT

In this paper global buckling (i.e., flexural, pure torsional, or flexural–torsional buckling) of thin-walled columns is discussed. The considered problem is the most basic one: the column is simply supported and subjected to a uniform concentric compressive force. The column's cross-section is an arbitrary open thin-walled cross-section. For the critical forces of this problem classical analytical solutions are known. In the presented research alternative formulae are derived on the basis of modeling the member as a set of flat plane elements (or strips). As it is found, the derivations can be carried out in various ways, among which eight options are considered. The resulted critical force formulae are briefly discussed in this paper. Extensive numerical studies are also completed; these studies are summarized in a companion paper.

1. Introduction

Analytical formula for the buckling of columns are known since Euler published his solution for flexural buckling in 1744, but torsional and flexural–torsional buckling can also be regarded as a classical problem the solution of which is dated back to the first half of the 20th century. These solutions, now, are part of the engineering education and can be found in textbooks, e.g. in [1].

An important and characterizing feature of these solutions is that they use a one-dimensional model for the column which will be referred here as *beam model*. Using a beam model, the structure (e.g., frame or truss) is represented by lines (most frequently straight lines), and to each point of the lines cross-sections are assigned, characterized by the cross-sectional properties. In other words, in the classical beam model it is a priori assumed that the whole displacement field of a beam or column can be expressed by the displacements of a reference line (which reference line is most frequently and conveniently the line defined by the mass centers of the cross-sections). The practical usefulness of the beam model is unquestionable: beam model was the only model for frames and trusses for a long time, but even today it is the model on which most frame and truss design is normally based. Also, the above-mentioned closed-formed solutions for column buckling, perhaps, would not have existed without the beam model.

It is also important to observe, however, that the beam model has its own limitations which prevent it to correctly model certain phenomena. More exactly, the classical beam model is essentially unable to consider any phenomena which involve the change of the cross-section (e.g., local or distortional buckling, flange curling), or any localized phenomena which affects primarily only a small part of a cross-section (e.g., the effect of small holes). Therefore, more sophisticated models (which are also more complicated) might be useful even in case of beams or columns. And due to the recent development of computers and numerical methods, the application of more sophisticated models does not mean a major difficulty any more, since finite element software packages are easily available and widely used even in the everyday engineering practice.

Thin-walled members are certainly among the cases where more sophisticated models might be useful, since local plate buckling, distortional buckling, interacted buckling modes, flange curling, shear lag, all significantly influence the behavior of a thin-walled beam or column, while these phenomena are all out of the scope of the classical beam model. A possible way for the analysis is the application of the finite strip method (FSM) or the finite element method (FEM) with using shell finite elements.

The common characterizing feature of FSM and shell FEM is that the thin-walled member is modeled by flat plane elements (in the simplest case rectangular elements), to which thicknesses are assigned, and each element is able to sustain membrane stresses (due to in-plane forces) and bending stresses (due to out-of-plane actions). Depending on how these membrane and bending stresses are calculated, and depending on the practical realization of the assumptions, various FSM and FEM applications are possible and existing, still, recognizing the above-mentioned important common feature (and for the sake of simplicity), this type of structural model hence will be referred as *shell model*.

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Of course, the shell model has its own limitations, too, since those phenomena where the change of a plate thickness (due to the effect of forces, for example) would be important, cannot be handled correctly. More importantly the model for the connection of the flat parts is approximate (e.g., corners). Nevertheless, shell models provide appropriate accuracy for most of the phenomena, which statement is especially true for cold-formed steel members with constant wall thickness and high width-to-thickness ratios.

Once shell model is used for a thin-walled member, it is a natural idea to use it for the calculation of flexural (F), torsional (T), flexural–torsional (FT) or lateral–torsional (LT) buckling, too, which buckling modes will be referred here as *global buckling*. It is observed, however, that by using a shell model for global buckling, perfect coincidence with the classical analytical solution is hardly achieved, even though a fine enough mesh is used and even though in case of a single member the size of the problem cannot cause numerical inaccuracy.

The paper can be regarded as the extension of earlier works of the author (see [2,3]), and has the aim to explore the reasons of the existing differences between beam model and shell model global column buckling solutions. Analytical solutions (i.e., formulae) are derived on the basis of shell model assumptions which can conveniently be compared to classical beam-model-based analytical formulae. Since shell model may be realized in a number of more-or-less different ways, the derivation of shell-model-based buckling solution is carried out for various options. The resulted formulae then highlight the importance of these options, and explain most of the differences between beam model and shell model solutions. Although the differences are small in many practical applications, the shell-model-based formulae point out those cases where significant differences between various solutions are possible. Moreover, it is believed that the presented research contributes in the deeper understanding of global column buckling.

This paper concentrates on the derivations of analytical solutions for shell-model-based critical forces. The theoretical considerations, however, are supplemented by extensive numerical studies in which various options of beam and shell-model-based solutions (by using various tools, including well-known finite element software) are analyzed and compared to one another. The numerical studies are summarized in a companion paper (see [4]). The results of the numerical studies justify the here-derived shell-model-based formulae, demonstrate the differences, as well as highlight the importance of a clear global buckling definition.

2. Derivation of shell model formula for global buckling

2.1. General

The aim is to derive an analytical expression for the critical force of a thin-walled column with arbitrary open cross-section. An illustration of the member as well as the applied global (*X*,*Y*,*Z*) and local (*x*,*y*,*z*) coordinate systems are presented in Fig. 1.

The applied basic mechanical assumptions intend to imitate those of a FSM or shell FEM solution. More exactly, the assumption system closely follows that of the semi-analytical FSM, as implemented in CUFSM [5] (see also [6,7]). However, some additional options are also considered. In case of shell FEM, since various implementations exist, the presented solution can be regarded as an approximation, though it is believed that the results well represent many FEM implementations, especially those which use classical plate bending theory.

For the analyzed member it is assumed that: (i) the analyzed member is a column, (ii) the column is prismatic, (iii) the column is supported by two hinges at its ends, (iv) the column is loaded by a compressive force (uniformly distributed along the cross-section), (v) its material is linearly elastic, and (vi) it is free from imperfections (residual stresses, initial deformations, material inhomogenities, etc.).

As far as boundary support conditions are concerned, the applied longitudinal shape functions correspond to globally and locally pinned and free to warp support conditions. More precisely, for both column ends: (i) local transverse translations (i.e., in the *x* and *z* directions) are restrained, which also means that global transverse translations (i.e., in the *X* and *Z* directions) are restrained, (ii) translations in the *y* or *Y* direction can freely occur, i.e., the cross-section warping is allowed, (iii) local twisting rotations of the strips are restrained, which also means that global twisting of the cross-section is restrained, (iv) and finally local rotations about the strips' local *x*-axis can freely occur, as well as global rotations about global *X*- and *Z*-axis can freely take place.

For the member deformation and displacements we assume that (i) the member is modeled by 2D surface elements, hence referred also as *strips*, (ii) in-plane (membrane) and out-of-plane (plate bending) deformations are allowed, (iii) for the in-plane behavior a classical 2D stress state is considered, (iv) for the out-of-plane behavior a classical Kirchhoff plate is considered, and (v) displacements are constrained to global buckling mode (as defined in Section 2.2).

For the derivation of the formula the energy method is used. The total potential energy of the member is expressed, and critical force is searched by utilizing that in equilibrium the total potential energy is stationary.

2.2. Definition for global buckling

In classical beam-model-based solutions for column buckling three displacement degrees of freedom (DOF) are assumed: two transverse translations and the rotation about the longitudinal axis. This also suggests that cross-section distortion does not take

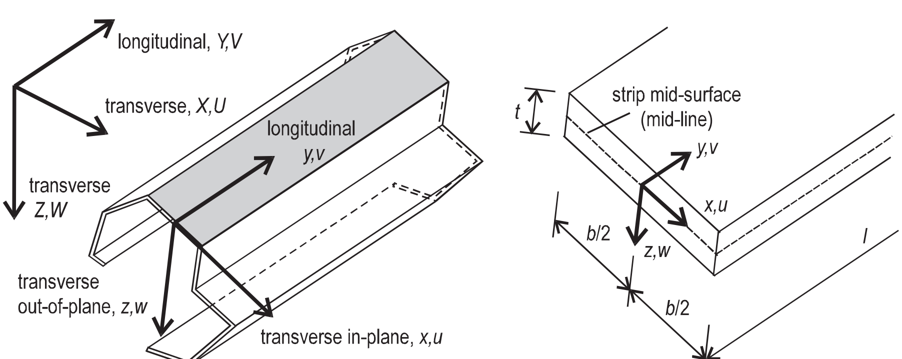

Fig. 1. Coordinate-systems, basic terminology.