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Elastic buckling of cylindrical pipe linings with variable thickness encased in rigid host pipes

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ABSTRACT

Cured-in-place plastic pipe linings are widely used in the rehabilitation of deteriorated rigid pipelines. Generally, the thickness of the pipe lining is assumed to be constant for simplification during the previous analysis. However, the thickness of the pipe lining may be a variable due to corrosive liquids or gases in service. This paper develops an analytical solution for the elastic buckling of cylindrical pipe linings with variable thickness subjected to external hydrostatic pressure. In addition to the analytical solution based on the minimum potential energy theory, a numerical analysis using the finite element method (FEM) is also performed for comparison. The FEM results agree well with the analytical solutions. Both the FEM results and the analytical solutions are discussed and compared with the models presented by other authors. When the thickness of pipe lining is constant, our analytical solutions can be simplified to Glock's typical solutions.

1. Introduction

In recent decades, the cost-efficient trenchless rehabilitations of damaged pipes and sewers have developed rapidly as a conventional method. In fact, most of the damaged pipes and sewers are rigid enough to carry the soil and surcharge loads, but renovation is required due to the leakage of damaged pipes. One solution is to embed thin-walled polymeric (steel) pipe linings to the deteriorative sewer pipes [1,17]. Pipe linings are often installed in host pipes that lie below the ground water level, and so the linings are under external hydrostatic pressure, which is transmitted through the leaking cracked pipes. In this case, the loads (e.g. soil loads) surrounding the pipe are carried by the existing rigid host pipe, and the pipe lining carries only the pressure from the external hydrostatic pressure. The buckling of cylindrical pipe lining under external pressure has been extensively studied. Generally, the pipe lining is considered infinitely long so that we can treat it as a ring. Therefore, the buckling of pipe lining can be viewed as a plane strain problem [2,3].

Timoshenko and Gere [4] studied the stability of a thin unconstrained circular ring under external hydrostatic pressure and proposed the following non-dimensional expression:

$$\frac{P_{cr}(1-\mu^2)}{E} = 2\left(\frac{h}{2R}\right)^3 \tag{1}$$

where P_{cr} is the critical buckling pressure, E is Young's modulus of the pipe lining, μ is Poisson's ratio, R is the cylinder radius, and h is the wall thickness.

Encased pipe linings may collapse at a higher critical pressure than unconstrained linings [5–8]. Cheney [9] used small-deflection linear theory to study the hydrostatic buckling of a ring encased in a rigid cavity. Cheney assumed that the wall of the cavity moves inward with the ring, resisting outward movement but not inward movement. Glock [10] and Omara et al. [11] studied the stability of a thin ring encased in a rigid cavity by the minimum potential energy method. Unlike Cheney's model, Glock's model assumed that the cavity does not move inward with the ring. Glock's model is shown in Fig. 1, and the critical pressure satisfied the following formula:

$$\frac{P_{\text{Glock}}(1-\mu^2)}{E} = \left(\frac{h}{2R}\right)^{2.2} \tag{2}$$

Boot [12] extended Glock's solution by considering the effect of a gap between the lining and its host pipe and reported implicit analytical expressions for the buckling pressure. Omara et al. [13] studied the instability of thin pipes in an oval rigid cavity using analytical models and experiments.

El-Sawy and Moore [14] conducted an elastic finite-element analysis of liners, taking into account the initial gap between the host pipe and the lining. Based on their numerical results, El-Sawy and Moore proposed the following empirical regression solution to express the buckling pressure of elastic pipe linings:

$$\frac{P_{cr}(1-\mu^2)}{E} = 2\left(\frac{h}{2R}\right)^3 \frac{25+350(h/R)+315(g/R)}{0.15+65(h/R)+350(h/R)^2+145(g/R)} \tag{3}$$

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When $g=0$, the pipe lining and its host pipe are tightly fitted, so Eq. (3) can be transformed to

$$\frac{P_{cr}(1-\mu^2)}{E} = 2\left(\frac{h}{2R}\right)^3 \frac{25+350(h/R)}{0.15+65(h/R)+350(h/R)^2} \quad (4)$$

Madryas and Szot [15] considered the effects of gap, ovality, wavy imperfections as well as combinations of all three imperfection types with the two-dimensional (2D) finite-element method (FEM). As a result, a regression formula was proposed to estimate the buckling pressure. Additionally, Vasilikis and Karamanos [16] developed a plastic hinge mechanism and obtained a closed-form expression to illustrate the post-buckling of the pipe lining. Recently, El-Sawy and Sweedan [17] investigated the effect of local wavy imperfections on the elastic stability of cylindrical linings using the three-dimensional (3D) finite-element model, and derived empirical formulas that agree with Glock's model.

All of the above works assumed that the thickness of the shells (rings) was constant. In some cases, thin-walled cylindrical shells may have variable thickness. Bai and Hauch [18,19] proposed analytical solutions for the collapse capacity of corrosive pipes by extending the existing analytical solutions, and the corresponding model is shown in Fig. 2(a). Xue and Fatt [20] studied the buckling of a non-uniform, long cylindrical shell subjected to external hydrostatic pressure, illustrated in Fig. 2(b). The exact solutions of elastic buckling pressure were presented, and the analytical solutions agreed with their finite-element predictions.

These non-uniform models are local imperfections along the circumferential direction. The thickness of the pipe lining may vary over the entire circumference due to the following circumstances:

(1) The liquid level floats up and down, so the unfilled portion may be impaired by corrosive gases mixed with air as shown in Fig. 3.
(2) Manufacturing errors in the pipe linings may occur.
(3) When the linings are cast in situ, the thickness may vary due to gravity.

This paper focuses on the elastic buckling of cylindrical pipe linings with variable thickness subjected to external hydrostatic pressures. At the beginning, analytical solutions are presented based on the minimum elastic potential principle. A numerical analysis shows how the thickness parameters affect the buckling modes. We compare the FEM predictions with the presented analytical solutions and the available predictions and find that the analytical solutions are almost the same as the numerical results in their predictions of the elastic buckling pressure of pipe linings encased in rigid host pipes.

2. Basic equations

This research is based on the corrosion of lining's inner surface. It is shown that the outer radius of the lining (R_2) is constant in Fig. 4. The thickness of the ring in the circumferential direction is assumed to be of the form

$$h = h_0\left(1-m+m\frac{\phi}{\pi}\right); \quad 0 \le \phi \le \pi \quad (5)$$

where the crown of the ring (point C) is the position for $\phi=0$ and the bottom (point B) is $\phi=\pi$, and h_0 is the thickness of the ring at $\phi=\pi$. The thickness variable parameter m is positive, and m varies from 0 to 0.5.

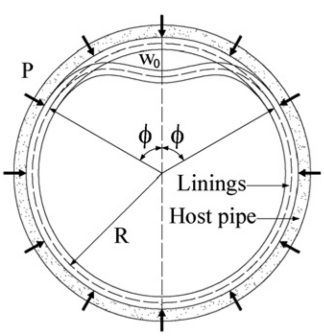

Fig. 1. Glock's model of buckling ring.

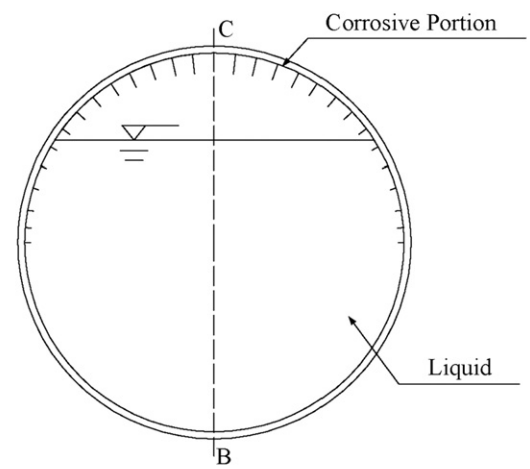

Fig. 3. Pipe lining's variable thickness due to corrosion.

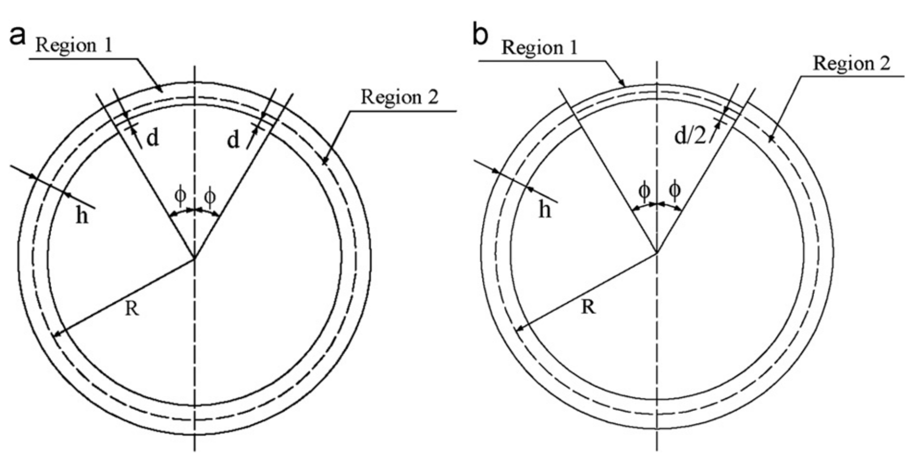

Fig. 2. Models of corrosive pipe linings: (a) Bai and Hauch's model of buckling ring and (b) Xue and Fatt's model of buckling ring.