Thin-Walled Structures 51 (2012) 121–129

Contents lists available at SciVerse ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Geometrically non-linear generalised beam theory for elastoplastic thin-walled metal members

Rodrigo Gonçalves [a,*], Dinar Camotim [b]

[a] UNIC, Departamento de Engenharia Civil, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal
[b] ICIST/IST, Departamento de Engenharia Civil e Arquitectura, Universidade Técnica de Lisboa, Av. Rovisco Pais, 1049-001 Lisbon, Portugal

ARTICLE INFO

Article history:
Received 17 August 2011
Received in revised form
8 October 2011
Accepted 10 October 2011
Available online 9 November 2011

Keywords:
Generalised Beam Theory (GBT)
Thin-walled structures
Beam finite elements
Elastoplastic collapse

ABSTRACT

This paper presents the formulation and validation of a geometrically and physically (J_2 plasticity) non-linear Generalised Beam Theory formulation, intended to calculate accurate non-linear elastoplastic equilibrium paths of thin-walled metal bars and associated collapse loads. This formulation extends previous work (Gonçalves and Camotim, 2011) [1] by including the geometrically non-linear effects. The plate-like bending strains are assumed to be small (as in all GBT formulations), but the membrane strains are calculated exactly. Both stress-based and stress resultant-based GBT approaches are developed and implemented in a 3-node beam finite element. The stress-based formulation is generally more accurate, but the stress resultant-based formulation makes it possible to avoid numeric integration in the through-thickness direction of the walls. In order to show the potential of the proposed formulation and resulting finite element, several numerical results are presented and discussed. For validation purposes, these results are compared with those obtained with standard 2D-solid and shell finite element analyses.

1. Introduction

Generalized Beam Theory (GBT) constitutes an extremely efficient and versatile tool to solve a wide range of structural problems involving thin-walled bars. GBT was first proposed by Schardt more than 40 years ago [2,3],[1] but its dissemination and widespread application initiated only after the first publications in the english language [4,5]. Since then, various researchers have been working with GBT (e.g., [6–8]), helping establish it as an efficient alternative to the finite strip and shell finite element methods, with a considerable amount of the new contributions being authored by Camotim and co-workers (see, e.g., the state-of-the art reports [9–12]).

Concerning the application of GBT to elastoplastic materials, only a limited number of studies have been presented. The authors have proposed a formulation aiming at the calculation of local/distortional/global plastic bifurcation loads of metal thin-walled members [13–15]. This formulation is based on the linear stability analysis concept, where pre-buckling displacements are discarded, and only trivial pre-buckling uniaxial stress fields were dealt with. More recently, the authors presented a geometrically linear GBT formulation for the determination of non-linear elastoplastic equilibrium paths [1] where, besides the conventional (stress-based) GBT approach, a novel stress resultant-based formulation was proposed, which employs the Ilyushin yield function [16]. Although the stress-based formulation was found to be generally more accurate, it was also shown that the stress resultant-based formulation leads to significant savings from a computational point of view, namely (i) it does not require numeric integration in the through-thickness direction and (ii) makes it possible to enforce constraints to the stress resultant and work-conjugate strain field as in linear elastic GBT formulations, thus making it possible to discard some stress components and, more important, to reduce the number of admissible deformation modes (degrees of freedom).

In this paper, the previous formulation is extended to the geometrically non-linear setting, in order to make it possible to calculate accurately non-linear elastoplastic equilibrium paths and associated collapse loads. Both the standard stress-based and the stress resultant-based formulations are developed and implemented in a 3-node beam finite element. GBT formulations able to calculate geometrically non-linear *elastic* equilibrium paths were first developed by Miosga [17] and, more recently, by Silvestre and Camotim [18] and Simão [19]. In these formulations, geometrically non-linearity is included by adopting a total Lagrangian description and additively decomposing the strain terms into Green–Lagrange membrane strains and small-strain

* Corresponding author. Tel.: +351 21 2948580; fax: +351 21 2948398.
E-mail address: rodrigo.goncalves@fct.unl.pt (R. Gonçalves).

[1] See http://www.vtb.info for a list of references on GBT up to 2001.

0263-8231/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.tws.2011.10.006

bending. This approach is followed in the present paper although, in contrast with the previous formulations, the membrane strains are now calculated exactly, i.e., all non-linear membrane strain terms are included. Since small strains are assumed for the bending terms, the geometrically non-linear GBT formulations are generally restricted to the small-to-moderate displacement range but, as shown in the examples presented in this paper, the accuracy and computational efficiency of GBT is preserved within this range. For moderate-to-large displacements, it is better to adopt kinematic descriptions based on geometrically exact beam theories and employ rotation tensors to describe the cross-section rotation [20,21]. In order to illustrate the potential of the proposed formulation, several examples are presented and discussed. For validation and comparison purposes, 2D-solid/shell large displacement/small strain finite element analysis [22] results are employed.

Concerning the notation, (i) scalars are represented in *italic* and (ii) vectors and matrices in ***bold italic***. Partial derivatives are indicated by subscripts following a comma, e.g., $f_{,x} = \partial f / \partial x$. A virtual variation is denoted by δ and an incremental/iterative variation by Δ. Where no distinction is necessary, variations may be denoted by d.

2. Non-linear GBT fundamental equations and finite element implementation

2.1. Kinematic description

For the purpose of describing the configuration of a given thin-walled member, the reference (straight) configuration depicted in Fig. 1 is employed, where x, y and z are wall local axes along the longitudinal, cross-section mid-line and thickness directions, respectively. The current configuration of the member is mapped by the displacement vector $\boldsymbol{U}(x,y,z)$ which, for a given wall, is expressed as

$$\boldsymbol{U}(x,y,z) = \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix} = \begin{bmatrix} u(x,y) - z w_{,x}(x,y) \\ v(x,y) - z w_{,y}(x,y) \\ w(x,y) \end{bmatrix}, \tag{1}$$

where u, v, w are the wall mid-surface displacement components along x, y and z, respectively, given by

$$u(x,y) = \overline{\boldsymbol{u}}^t(y)\boldsymbol{\phi}_{,x}(x),$$

$$v(x,y) = \overline{\boldsymbol{v}}^t(y)\boldsymbol{\phi}(x),$$

$$w(x,y) = \overline{\boldsymbol{w}}^t(y)\boldsymbol{\phi}(x), \tag{2}$$

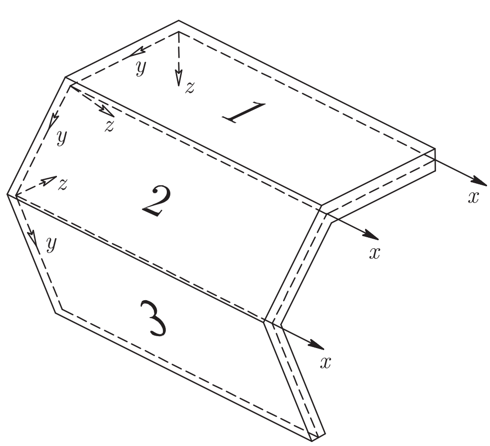

Fig. 1. Arbitrary thin-walled member geometry and local coordinate systems.

with (i) the column vectors $\overline{\boldsymbol{u}}(y)$, $\overline{\boldsymbol{v}}(y)$, $\overline{\boldsymbol{w}}(y)$ containing the GBT deformation mode displacement components along x, y and z, respectively, calculated from the "GBT cross-section analysis" (see, e.g., [3,23]), and (ii) the column vector $\boldsymbol{\phi}(x)$ containing their amplitude functions along the beam length. Note that the classic GBT description is preserved, namely:

(i) The displacements of material points located outside the wall mid-surface ($z \neq 0$) are obtained from Kirchhoff's plate theory assumption in the small displacement range. This simplification is not too restrictive, since it only affects the bending terms (not the membrane ones), which will be calculated assuming also small displacements, as mentioned previously. On the other hand, it has the advantage of making the formulation insensitive to plate-like shear locking.

(ii) The warping displacements u are associated with $\boldsymbol{\phi}_{,x}$ rather than with $\boldsymbol{\phi}$, in order to make it possible to enforce null membrane shear strains in the geometrically linear GBT formulation ($\gamma^M_{xy} = 0$, i.e., Vlasov's assumption). This obviously does not constitute a limitation for the geometrically non-linear formulation and is, therefore, adopted. In alternative, u could be associated with $\boldsymbol{\phi}$, which has the advantage of lowering the continuity requirement for the amplitude functions of the deformation modes involving warping displacements. However, the continuity requirement is still maintained for the modes with $w \neq 0$, due to Kirchhoff's assumption. A complete discussion of this matter was presented in [1].

By incorporating (2) into (1), one obtains

$$\boldsymbol{U}(x,y,z) = \underbrace{\overline{\boldsymbol{\Xi}}_{\boldsymbol{U}}(y)\begin{bmatrix} \boldsymbol{\phi}(x) \\ \boldsymbol{\phi}_{,x}(x) \end{bmatrix}}_{\overline{\boldsymbol{U}}} + z\boldsymbol{\Xi}_{\boldsymbol{U}}(y)\begin{bmatrix} \boldsymbol{\phi}(x) \\ \boldsymbol{\phi}_{,x}(x) \end{bmatrix}, \tag{3}$$

where $\overline{\boldsymbol{U}}$ is the displacement vector of the wall mid-surface and the auxiliary matrices read

$$\overline{\boldsymbol{\Xi}}_{\boldsymbol{U}}(y) = \begin{bmatrix} \mathbf{0} & \overline{\boldsymbol{u}}^t \\ \overline{\boldsymbol{v}}^t & \mathbf{0} \\ \overline{\boldsymbol{w}}^t & \mathbf{0} \end{bmatrix}, \quad \boldsymbol{\Xi}_{\boldsymbol{U}}(y) = -\begin{bmatrix} \mathbf{0} & \overline{\boldsymbol{w}}^t \\ \overline{\boldsymbol{w}}^t_{,y} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{4}$$

If the beam initial configuration is not straight, it is useful to define the displacements between the reference and initial configurations through

$$\boldsymbol{U}_0(x,y,z) = \underbrace{\overline{\boldsymbol{\Xi}}_{\boldsymbol{U}}(y)\begin{bmatrix} \boldsymbol{\phi}_0(x) \\ \boldsymbol{\phi}_{0,x}(x) \end{bmatrix}}_{\overline{\boldsymbol{U}}_0} + z\boldsymbol{\Xi}_{\boldsymbol{U}}(y)\begin{bmatrix} \boldsymbol{\phi}_0(x) \\ \boldsymbol{\phi}_{0,x}(x) \end{bmatrix}, \tag{5}$$

and the column vector $\boldsymbol{\phi}_0(x)$ contains the amplitude functions required to describe the initial configuration.

Finally, the displacements between the initial and current configurations are given by

$$\hat{\boldsymbol{U}}(x,y,z) = \boldsymbol{U} - \boldsymbol{U}_0 = \underbrace{\overline{\boldsymbol{\Xi}}_{\boldsymbol{U}}(y)\begin{bmatrix} \hat{\boldsymbol{\phi}}(x) \\ \hat{\boldsymbol{\phi}}_{,x}(x) \end{bmatrix}}_{\hat{\overline{\boldsymbol{U}}}} + z\boldsymbol{\Xi}_{\boldsymbol{U}}(y)\begin{bmatrix} \hat{\boldsymbol{\phi}}(x) \\ \hat{\boldsymbol{\phi}}_{,x}(x) \end{bmatrix}, \tag{6}$$

with $\hat{\boldsymbol{\phi}}(x) = \boldsymbol{\phi}(x) - \boldsymbol{\phi}_0(x)$.

2.2. Strains

The relevant strain components for each wall are grouped in vector $\boldsymbol{E}^t = [E_{xx}\ E_{yy}\ 2E_{xy}]$. An additive decomposition $\boldsymbol{E} = \boldsymbol{E}^M + \boldsymbol{\varepsilon}^B$ is assumed, where $(\boldsymbol{E}^M)^t = [E^M_{xx}\ E^M_{yy}\ 2E^M_{xy}]$ are Green–Lagrange membrane strains and $(\boldsymbol{\varepsilon}^B)^t = [\varepsilon^B_{xx}\ \varepsilon^B_{yy}\ \gamma^B_{xy}]$ are small bending strains.