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A new approach for thin-walled member analysis in the framework of GBT

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ABSTRACT

A new approach is illustrated for the cross-sectional analysis to be performed in the context of the Generalised Beam Theory (GBT). The novelty relies in formulating the problem in the spirit of Kantorovich's semi-variational method, namely using the dynamic modes of an unconstrained planar frame as in-plane deformation modes. Warping is then evaluated from the post-processing of these in-plane modes, thus reversing the strategy of the classical GBT procedure. The new procedure does not require several steps of the classical algorithm for the determination of the conventional modes, in which bending, shear and local modes are evaluated separately, and is applicable indifferently to open, partially-closed and closed sections. The efficiency and ease of use of the method are outlined by means of two examples, aimed to describe the linear–elastic behaviour of thin-walled members.

1. Introduction

The Generalised Beam Theory (GBT) is a powerful tool for the elastic and buckling analysis of thin-walled members (TWM). The basic ideas of the method have been originally proposed by Schardt [1,2] and disseminated in English by Davies and co-workers [3–7] while its large diffusion in the scientific community is due to the strong impulse given to it in the last decade by Camotim and co-workers [8–27]. They generalised the method to include new aspects, not present in the original formulation, and combined it with a FE approach [8–10], so that GBT is now applicable to anisotropic members [11], branched open sections [12], closed or partially closed sections [13], circular sections [14], non-standard support conditions [15,16], frames of TWM [17], buckling problems [10,13,14,18–22], linear dynamic problems [23,24], and post-buckling problems [25–27]. In recent years, Adany and Schafer [28,29] and Casafront et al. [30] applied the principles at the basis of the GBT to reduce the number of freedoms required in performing buckling investigations using the finite strip method and finite element method, respectively.

As well-known, GBT considers a TWM as an assembly of (generally, but not necessarily, flat) thin plates, free to bend in the plane orthogonal to the member axis. Thus, GBT accounts for deformable cross-sections, differently from the classical Vlasov theory [31] in which the cross-section keeps its original shape. The basic idea of the method consists in describing the displacement field of the TWM as a linear combination of assumed 'deformation modes' of the cross-section (including in-plane and warping components), and 'amplitude modes', which are unknown functions depending on the axial coordinate. A variational principle, as the virtual work equation, provides the weak formulation of the problem, leading to a system of ordinary differential equations in the unknown amplitudes, with the relevant boundary conditions. These equations, equal in number to the deformation modes considered, generalise the classical Vlasov beam theory, where the latter can be described using four amplitude functions, each associated to a rigid motion of the section, namely three translations (two flexures and an extension) and one rotation (torsion) around the shear centre. Even if not initially noted by Schardt, this approach falls within Kantorovich's semi-variational method, aimed at reducing the dimensionality of a problem through a technique of partially-assumed modes. Thus, in the case of the GBT, a three-dimensional continuous problem is transformed into a vector-valued one-dimensional problem. In particular, the GBT method consists of two phases: (1) the choice of the deformation modes, referred to as 'cross-sectional analysis', and (2) the solution of the amplitude equations, denoted as 'member analysis'. The fundamental step of this method relies on its ability to determine a suitable set of deformation modes.

A brief overview of the essence of the classical GBT is provided in the following, partly for readers not fully familiar with this approach, to highlight the procedure involved in the evaluation of the conventional deformation modes and to better outline the contribution of the proposed work. Throughout this paper and consistently with the definitions provided in [8], conventional modes are assumed to include the rigid-body modes, the distortional ones, the local (bending) ones complemented, when dealing with closed sections, with a shear mode to depict the case of pure torsional shear flow.

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The first step of the GBT formulation is to discretise the cross-section of a member in plate segments (with nodes inserted at their ends) along which linearly varying warping functions are assumed to exist. By following the methodology commonly used with the direct stiffness method, unit warping displacements are applied to one node at the time, while keeping all the remaining ones equal to zero, thus creating a set of 'warping modes'. The corresponding in-plane tangential displacements of the segments are determined for each of these modes based on the zero-shear Vlasov condition (valid for open cross-sections). This procedure leads to a loss of compatibility at the 'natural nodes' (i.e. at the corners of the plates). A rigid-body kinematic problem is then solved to restore the compatibility of the translations by applying normal displacements and rotations to each plate segment. After this, the force method is used to restore compatibility of the rotations at the natural nodes, while treating the plate segments as (deformable) continuous members and assuming their corners restrained by pinned supports. With this procedure, a basis of linearly independent modes (with number equal to the number of nodes of the section) is created. This set of 'warping modes', however, is not exhaustive, since it is associated to deformations in which the nodes translate in the cross-section plane. 'Bending modes', instead, in which the plate deforms by leaving the corners (practically) immovable and without warping, cannot be determined with this procedure. The way that GBT addresses this problem is to restart a new calculation in which unit displacements are assigned, this time normal to the plate segments, at 'non-natural' intermediate nodes (i.e. *not* at the corners), followed, also in this instance, by the solution of the elastic problem of a continuous member. This second set of modes, however, does not include deformations of *closed* sections in which the displacements are essentially tangential, triggering shear deformations of the plates, and therefore referred to as 'shear modes'. To determine these latter ones another *ad hoc* procedure needs to be applied, which induces unit tangential displacements to each segment of the cross-section belonging to closed cells (based on the considerations that membrane shear strains are negligible in segments included in open branches).

The different deformation modes constructed, based on the procedures previously outlined, are *local-type* in nature because they involve nontrivial displacements only in a few adjacent segments. Remaining consistent with their derivations, these modes could be used in this form, in a similar manner as, for example, the finite strip approach is applied to TWM analysis or splines are used to describe extended functions (e.g., [32–36]). Such a *local-type* representation, however, is not convenient, if one desires to use few significant modes to capture the main structural behaviour. Therefore, a change of basis is then performed in the classical GBT to obtain *global type* deformation modes. This is obtained as the eigenvectors of a properly chosen eigenvalue problem, able to simplify the amplitude equations.

It is in authors' opinion that an unified procedure for the determination of the conventional deformation modes would contribute to a wider diffusion and use of the GBT approach. In this context, a new version of the method is proposed here, which, in the spirit of the semi-variational method selects the conventional deformation modes directly defined on the whole domain and chosen as the eigenvectors of a positive semi-definite eigenvalue problem. The free dynamics of the unconstrained planar frame, represented by the plate segments forming the cross-section placed at their mid-lines, is chosen as the eigenvalue problem. Since the frame is free in its plane, it possesses rigid motions that account for the Vlasov beam theory, and flexural modes which account for deformation modes. Once the planar modes are determined, e.g. using a standard finite element analysis, even performed with a commercial software, the cross-sectional analysis is completed by evaluating the corresponding warping displacements based on conditions enforced on the shear strain. Among these, the purely extensional mode appears as an arbitrary quantity rising from integration. In this way, the strategy used by the classical theory is reversed, in the sense that in plane-components are evaluated first, and warping components successively.

This paper starts by briefly recalling the basis of the GBT, limiting the description to the first-order analysis. This is followed by the new proposed cross-sectional analysis and its ease of use is outlined by means of two applications on simply-supported TWM. For clarity, the procedure proposed for the calculation of the warping displacements, starting from the in-plane ones, is detailed with an example in Appendix A.

2. Basis of the GBT approach

A generic thin-walled-member is considered with (a) open, (b) closed or (c) partially-closed cross-section formed with flat plates (Fig. 1). The displacement field $\mathbf{u}(s,z)$ of an arbitrary point P(s,z) lying in the mid-plane of the section thickness is expressed as (Fig. 2):

$$\mathbf{u}(s,z) = u(s,z)\mathbf{e}_s(s) + v(s,z)\mathbf{e}_y(s) + w(s,z)\mathbf{e}_z(s) \quad (1)$$

where s is the curvilinear abscissa (if necessary defined on several branches) along the section mid-line $\mathcal{C}$, z is the coordinate along the member axis, $\mathbf{e}_s(s)$, $\mathbf{e}_y(s)$ and $\mathbf{e}_z(s)$ are unit vectors in the tangential, normal and bi-normal directions at the abscissa s, respectively, and $u(s,z)$, $v(s,z)$ and $w(s,z)$ are the displacement components in the same triad.

2.1. Displacement and strain fields

In the framework of the GBT, and making use of Kantorovich's semi-variational method, the displacement components of points

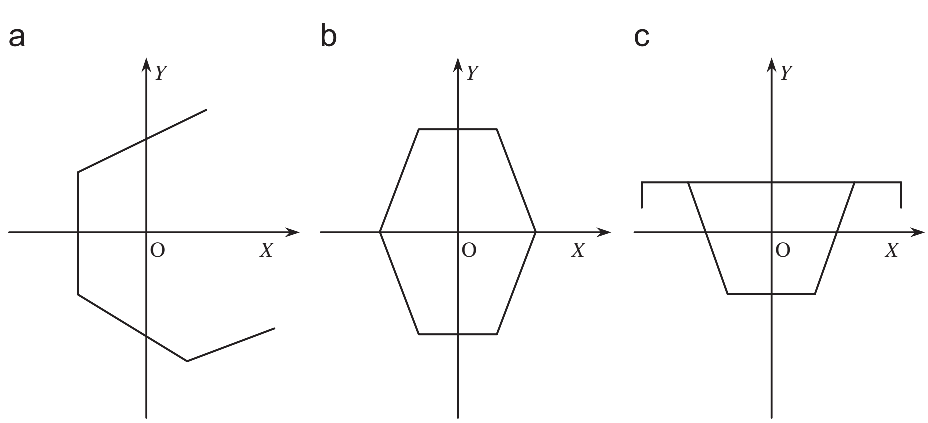

Fig. 1. Generic thin-walled cross-sections: (a) open cross-section; (b) closed cross-section; and (c) partially-closed cross-section.