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Research Article

Learning mathematics without a suggested solution method: Durable effects on performance and brain activity

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ABSTRACT

A dominant mathematics teaching method is to present a solution method and let pupils repeatedly practice it. An alternative method is to let pupils create a solution method themselves. The current study compared these two approaches in terms of lasting effects on performance and brain activity. Seventy-three participants practiced mathematics according to one of the two approaches. One week later, participants underwent fMRI while being tested on the practice tasks. Participants who had created the solution method themselves performed better at the test questions. In both conditions, participants engaged a fronto-parietal network more when solving test questions compared to a baseline task. Importantly, participants who had created the solution method themselves showed relatively lower brain activity in angular gyrus, possibly reflecting reduced demands on verbal memory. These results indicate that there might be advantages to creating the solution method oneself, and thus have implications for the design of teaching methods.

1. Introduction

One of the fundamental cognitive skills an individual has to learn to master during development is the ability to reason logically with numbers. In fact, the ability associated with mathematical understanding during school age has been found to be highly predictive of success later in life (e.g. [1]) while poor mathematical skills can have negative consequences for the individual (e.g. [2,3]). Not surprisingly, mathematics is prioritized as a core subject in all school systems, from kindergarten to college, and countries' educational qualities are consistently evaluated and compared not least on the basis of pupils' mathematical performance (e.g. TIMSS and PISA international surveys). Recently, the neurosciences have witnessed an increase in the number of studies targeting learning of arithmetics (for one review, see e.g. [4]).

How can an educational system assure that mathematics is being taught in a way that most efficiently promotes mathematical learning? This is an area of extensive debate [5–7]. What has been observed in detailed analyses of mathematics textbooks and curriculums is that one dominant mathematical teaching method centers on presenting typical task types and then give suggestions for solution methods (for examples from Sweden, see: [8,9], and from the US: [7]). These suggestions for solutions, commonly in the form of algorithmic templates (e.g. rules, methods, solved example tasks: [10]), are then typically subjected to extensive repeated practice, for example via practice tasks throughout a book chapter. A typical introductory example task in a chapter on percentages could read: "Of 80 students finishing grade nine, 16 applied for the natural science upper secondary program. How many percent of the students was that?" This is then followed by a template solution and the correct answer: "Proportion of applicants: 16/80 = 0.20 = 20% Answer: 20% of the students applied for

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the natural science program." Finally, this is usually followed by many practice tasks that are isomorphs to the introductory task, for example: "At a traffic control outside a school it was found that 84 cars out of 400 drove too fast. How many percent was that?"

Such teaching methods are guaranteed to lead to learning in the short term but conceptually, they appear to have much in common with 'rote learning': the process of learning something by repeating it until you remember it rather than by understanding the meaning of it (cf. Oxford Advanced Learners Dictionary). However, in spite of being short-term efficient there are data indicating that teaching based only on such methods fail to enhance students' long-term development of conceptual understanding [7]. Throughout this paper, we will refer to mathematical teaching methods of this kind as methods inviting Algorithmic reasoning (AR) [10].

As an alternative, it has been suggested that encouraging the individuals to create a solution method themselves should be superior for promoting mathematical learning, compared to explicitly presenting the solution method and invite extensive repeated practice with it [5]. This suggestion has been further specified by Lithner and colleagues by designing practice tasks inviting Creative Mathematically founded Reasoning (CMR) [8,10,11]. To compare with the example above, a task inviting CMR would include the same type of task, for example: "At a traffic control outside a school it was found that 84 cars out of 400 drove too fast. How many percent was that?"—but would not be preceded by the solved introductory task and template solution. Moreover, instead of being followed by many practice tasks, a task inviting CMR would instead be followed by explicit encouragement to create a solution method (e.g. a formula for the solution of the task). Jonsson et al. [11] have recently demonstrated that practice tasks designed to invite CMR might have superior effects on performance compared to tasks designed to invite AR.

The purpose of this study was to further compare these two kinds of practice tasks—designed to invite AR and CMR, respectively, both in terms of performance as well as in terms of brain activity. Participants first trained in an environment where they solved numerical tasks *with* given solution methods (AR), or solved numerical tasks *without* given solution methods (CMR). One week later they were tested on similar numerical tasks without given solution methods while being scanned with functional magnetic resonance imaging (fMRI), which allowed comparing the two teaching methods in terms of their effects on mathematical performance as well as on brain activity. A central question was whether these two kinds of practice tasks give rise to performance differences in the long-term [11].

Key to an environment designed to invite CMR rather than AR is that the solution method is not given but has to be self-generated [5,10]. Cognitive psychology research show that generating an answer compared to just reading it has large positive effects on long-term retention of that material, an effect known as the *generation effect* (e.g. [12]; see [13] for a review). This effect is related to the *testing effect*: repeated testing on a content for learning has stronger effects on long-term retention compared to repeatedly studying the same content (e.g. [14]). The generation effect has also been demonstrated with mental arithmetic [15,16]. For example, it has been shown that more answers to multiplication problems are remembered after previous practice on generating the answers compared to just reading the problem together with the answer [15]. Further, the benefit of generating the arithmetic solution has been shown to be larger for participants with low prior knowledge [16]. Even though the generation effect and the testing effect are empirical phenomena, with a wide range of potential theoretical explanations (see e.g., [13,17–20]), the demonstrated long-term performance benefits after self-generation are robust and compelling.

If participants trained in the CMR environment will have an easier time accessing their knowledge of a solution method during a later test, are there reasons to believe that this is manifested in relative differences in brain activity? To date, imaging studies of mathematics have in part focused on arithmetic tasks such as one- or two-digit multiplication, subtraction or addition tasks [4] or on task solving with algebra (see e.g., [21]) also for advanced algebra (e.g. [22]). Less is known about to what extent relative differences in brain activity observed in such tasks are also evident during less constrained and more general solution modes, as in, for example, creative mathematically founded reasoning.

One central aspect of practice effects in mental arithmetic that has gained much attention in imaging research is the shift as a function of practice from procedural calculation operations to retrieval of stored facts from memory (cf. [23]). Combining neuroimaging with multiplication tasks, for example, this shift has been observed to be mirrored by a relative shift in activity from frontal areas to parietal areas, in particular to the *left angular gyrus* (see e.g., [24,25]; for a review see [4]). Angular gyrus plays a key role in a model for number processing [26] and has been shown to be important for operations that in general require access to verbal memory of arithmetic facts, potentially supporting the verbal aspects of mental arithmetic. Thus, to the extent that participants trained in the CMR environment will require less effort to retrieve their knowledge of a solution method at the one-week follow-up test, we expect relatively *lower* activity in left angular gyrus in the CMR compared to the AR condition.

Finally, in this study we also investigated the potential role of individual differences in cognitive abilities of relevance for mathematical performance. It has previously been suggested that working memory is one potent predictor of mathematical achievement (e.g. [27]). Can individual differences in working memory explain variance in mathematical performance over and above potential effects of the different practice tasks? It has also been demonstrated that another potent predictor of mathematical competence is the acuity of the *Approximate Number System* (ANS: see e.g. [28,29]). The ANS is said to represent numerical magnitude in a non-symbolic mode. In order to investigate which cognitive abilities – if any – that explain variance in performance over and above the practice tasks, we included measures of *working memory* (Operation Span) and *ANS acuity*, as well as measures of *vocabulary* (SRB1) and *visuo-spatial processing and integration* (Raven's advanced matrices).

We hypothesized that ? (a) *creative mathematically founded reasoning* (CMR) will promote *better* performance at test one week after training than *algorithmic reasoning* (AR) [11], (b) CMR will translate into less engagement of left angular gyrus at test compared with AR, and (c) that working memory and ANS acuity will be significant predictors of individual differences in mathematical performance (independent of teaching method).

2. Methods

2.1. Participants

Seventy-three pupils and students participated in the study. The pupils ($n=40$, 24 males, 18–20 years old, $M_{age}=18.5$ years, $SD_{age}=0.60$) studied in their last year of the Swedish "gymnasium" (compares with upper secondary school/senior high school) and were enrolled in programs with a focus on natural sciences. The students ($n=33$, 24 males, 18–22 years old, $M_{age}=20.0$ years, $SD=1.2$) were enrolled in different engineering programs, in their first semester, and had all completed advanced mathematics courses during the gymnasium. All participants were right-handed and had normal or corrected-to-normal vision. Participants signed a written