Shape optimization of cold-formed steel columns

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A B S T R A C T

The objective of this paper is to demonstrate the application of formal optimization tools towards maximizing the compressive strength of an open cold-formed steel cross section. In addition, in the work presented here the cross section shape is not limited by pre-determined elements (flanges, webs, stiffeners, etc.), as is commonly required to meet the necessity of conventional code-based procedures for design that employ simplified closed-form stability analysis. Instead, by utilizing the finite strip method for stability analysis and the Direct Strength Method for the strength calculation, the full solution space of cold-formed steel shapes may be explored. In the analysis herein, a given width of sheet steel is allowed to be bent at 20 locations along its width, thus providing the ability to form nearly any possible shape. Three optimization algorithms are explored: the gradient-based steepest descent method and two stochastic search methods, genetic algorithms and simulated annealing. Compared with a standard cold-formed steel lipped channel the final optimized capacities are found to be more than double the original design. Steepest descent solutions are shown (as expected) to be highly sensitive to the initial guess, but they provide symmetrical and conceptually clean solutions. The stochastic search methods require significantly more computational capacity, explore the solution space more fully, and generate solutions that are largely insensitive to the initial guess. For long and intermediate length cold-formed steel columns the optimization methods identify two non-conventional alternative designs that maximize capacity. The future of this work lies in further integrating the optimization methods with additional manufacturing and construction constraints; for now, the method suggests several interesting alternative cross sections that are worthy of future study.

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1. Introduction

Recently, cold-formed steel (CFS) members have been used extensively in buildings as both structural (load-bearing) and non-structural (partitions and building envelope) members. CFS members have a typical thickness of approximately 1 mm and typical section depths from approximately 75 mm to 300 mm. One of the desirable features of CFS members is that they may be shaped (cold-bent) to nearly any open cross section that provides an efficient and economical solution. As a result, finding optimal shapes for CFS members from the vast geometry of possible designs is a problem of great interest.

In structural optimization a common engineering design objective is the minimization of weight while satisfying strength and serviceability constraints. On a member level this objective may be pursued by optimizing individual cross section profiles. The optimization framework may progress along two paths: algorithms based on formal mathematical programming, or algorithms based-on principles of stochastic search. The former are powerful and capable of identifying local optimum but are restricted to problems where design sensitivities, or derivatives (hence the term “gradient-based” methods) of performance metrics with respect to cross section design variables, are available. Stochastic search algorithms, on the other hand, do not require sensitivity information and thus may be applied to discrete problems, such as optimizing cross section selection from a database. The disadvantages are that they are heuristic trial-and-error methods, requiring a large number of analyses with no guarantee that an optimal solution will be found. Beyond the optimization framework the formulation of the objective function plays a critical role in the evaluation. If evaluation of the strength and serviceability constraints requires the use of typical Specification procedures (e.g., the main body of AISI-S100-07 [1]) then the full solution space cannot be explored and optimal designs are limited to shapes, elements and features (e.g., stiffener shape and size) that the Specification specifically addresses.

A number of researchers have proposed applying optimization methods to cold-formed steel cross section selection and design. Seaburg and Salmon [2] used a gradient-based (steepest descent) optimization framework to explore the dimensions of hat sections (basic topology of the hat fixed) using the AISI Specification [3] for
strength evaluation. Tran and Li [4] expressed the weight of a lipped channel beam as a function of cross section dimensions and solved the optimization problem using the trust-region method based on various failure modes from BS 5950–5 [5] and ENV 1993-1-3 [6]. Tian and Lu [7] optimized dimensions of channel columns with and without lips according to provisions of BS 5950–5 and compared the performance of cross sections found using intuition with those found using sequential quadratic programming. As for heuristic methods, Adeli et al. [8] developed a computational neural network model and found the local minima of weight for hat, I and Z cross section CFS beams according to AISI ASD [9] and LRFD specifications [10], the global optimum design of hat-shape beams [11], and space trusses with lipped channel sections [12] following the AISI ASD specification [9,13]. Lee et al. [14,15] used genetic algorithms to search for optimal channel cross section dimensions for cold-formed steel columns under axial compression and beams under uniformly distributed loads. Penalties were employed in the objective function for violating constraints from the AISI specification [16]. All of these authors designated a shape before optimization, and by adjusting the size of the cross section (depth, width, thickness, lip length, etc.) subjected to various constraints, they sought a minimum cross section area and thus minimized the weight.

Since cold-formed steel members are usually thin-walled, they are subject to local plate buckling, distortional buckling [17,18], and global (Euler) buckling. The goal of this work is to identify cross sections that maximize capacity of a member with a given length, coil width (i.e., cross section perimeter), and sheet thickness. Instead of the minimum weight for a given cross section shape, this work explores the topology more freely, and from the manufacturer’s viewpoint; what is the maximum strength for a given amount of material?

A key and challenging task in the optimization process is to compute the buckling strength of candidate designs with complex cross sections. Although specifications adopted in the aforementioned literatures are enacted in different countries, they are based on the well-developed, classical effective width method [16,17]. The primary idea of the effective width method is to reduce a plate under nonlinear longitudinal stress into a plate with effective width under constant stress. This procedure becomes difficult and cumbersome for complicated cross sections that may appear during the shape optimization process.

The Direct Strength Method (DSM), adopted by AISI in Appendix 1 [19], is capable of determining the nominal load \( P_{cr} \) for arbitrary geometry columns provided the user specifies the elastic instabilities: i.e., the critical load in local \( P_{cr}^{el} \), distortional \( P_{cr}^{dis} \), global buckling \( P_{cr}^{glob} \), and the load at yield \( P_y \). This enables expansion of the cross section design space as tools exist to compute these loads for an arbitrary cross section. In this work, we use the open source software package CUFSM, which employs the finite strip method, developed and updated by Schafer et al. [20,21] for finding \( P_{cr}^{el}, P_{cr}^{dis} \), and \( P_{cr}^{glob} \).

Previously, Lu [22] embedded CUFSM in a genetic algorithm to optimize Z-shape cross section dimensions following the effective width design of Eurocode 3 [6]. Kolcu et al. [23] utilized Mindlin–Reissner finite strips to compute the critical load \( P_{cr} \) of an unlipped channel column and maximized \( P_c \) by sequential quadratic programming. In a direct precursor to this work, conducted with the senior author, Liu et al. [24] used DSM and CUFSM and exploited Bayesian classification trees to find a cross section with largest \( P_{cr} \). The cross section was discretized into equal length strips and the turn-angles between adjacent elements were employed as design variables. The capability of Bayesian classification trees to decrease the solution space has long-term promise, but requires expert knowledge that may not always be available. In addition, the number of strips (turn-angle locations) was relatively coarse.

In this paper, the authors explore the coupling of optimization algorithms with DSM and CUFSM for the design of cold-formed steel columns. The optimization methods explored include a gradient-based steepest descent (SD) algorithm and stochastic search methods including genetic algorithms (GA) and simulated annealing (SA). Optimization of members with two different physical lengths is considered so that the impact of different controlling stability modes on the design and strength may be explored.

## 2. Formulation of optimization problem

The design objective in this paper is to maximize the capacity of a steel column, cold-formed from a fixed width coil of sheet steel, under uniform compression. The thin sheet of steel comprising the column is discretized into narrow strips of equal width along the longitudinal direction. Cross section designs are encoded into a vector of relative turn-angles defined for every two adjacent elements. Turn-angles are locations of potential folding during the roll forming process. The design variable is defined as a vector of turn-angles:

\[
\theta = [\theta_1, \theta_2, \ldots, \theta_n]
\]

where \( n \) is the number of elements in the cross section and the length of each element is the width of steel sheet divided by \( n \).

The turn-angle \( \theta_i \) is defined as the change in angle between the axes of elements \( i \) and \( i-1 \), measured counter-clockwise (Fig. 1(a)). In the case of \( \theta_1 \), the angle is measured from the global \( x \)-axis to the axis of element 1. Further, \( \theta_1 \) is used as a reference angle (all other angles are relative to \( \theta_1 \)) and thus held fixed leaving the independent design variables, denoted as \( \mathbf{x} = [\theta_2, \theta_3, \ldots, \theta_n] \). In the previous work of Liu et al. [24], the authors assumed cross section symmetry or anti-symmetry, limited turn-angles to variations of \( 45^\circ \), and fixed a number of the turn-angles to be zero (unfolded segments). This facilitated solution of the optimization problem but restricted the design space and thus potentially reduced the maximum achievable strength (the global optimum). The only restriction in this paper is that segment overlap is not permitted, as this is not physically possible.

![Fig. 1. Lipped channel section and definition of design variables.](image)