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On free vibration analysis of thin-walled beams axially loaded

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ABSTRACT

In this paper, a numerical–experimental study about natural frequencies of thin-walled beams axially loaded is presented. Moreover, the influence of axial load in the frequencies is studied. The equations of motion are based on Vlasov's theory of thin-walled beams, which were modified previously to include the effects of shear flexibility, rotatory inertia in the stress resultants. Moreover, a constant axial load is incorporated to the formulation, both in the time and frequency domain. The differential equations are shown to be particularly suitable for analysis in the frequency domain using a state variables approach. A numerical investigation is carried out to reveal the influence of the axial load in several boundary conditions. Finally, free vibration experimental tests are presented, which allow verify the theory presented in this paper and provide good quality data that can be used for checking the accuracy and reliability of different theories.

1. Introduction

Thin walled and open section beams are extensively used as structural components in different fields as civil, aeronautical and mechanical engineering. Natural frequencies of axially loaded beams are often required in helicopter and turbine blades design among other applications. For some practical applications, the effect of axial force on natural frequencies and mode shapes are more pronounced than those of the shear deformation and/or rotary inertia. Free vibrations of doubly symmetrical beams or beams with one axis of symmetry are widely studied, in general by using Bernoulli–Navier theory. However, results about doubly unsymmetrical beams are rather scarce in the literature, especially those related with experimental evidences. In this case, triple coupled flexural–torsional vibrations are observed.

The determination of natural frequencies and modes of vibration of undamped continuous beams and shafts is discussed in detail by Pestel and Leckie [1], who also describe the calculation of dynamic response to harmonic excitation. Ebner and Billington [2] employed numerical integration to study steady state vibrations of damped Timoshenko beams. Numerous other applications can be found in the literature concerning straight and curved beams, as well as arch and shell structures. On the other hand, the theory formulated by Vlasov [3] has been extensively used in the dynamic analysis of thin-walled open section beams. Ambrosini et al. [4,5], proposed a modified theory, which is based on Vlasov's formulation, but it accounts the effects of shear flexibility, rotatory inertia and non-uniform cross-section. This formulation, using the so-called state variables approach in the frequency domain, is suitable for efficient numerical treatment, which on account of generality and precision can be very useful in a variety of applications.

Several authors have contributed to theories that account for coupling between bending and torsion in beams. Tanaka and Bercin [6] extended the approach of Bishop et al. [7] to study triply coupled uniform beams using Mathematica. Prokic [8] presented the five governing differential equations for coupled bending–torsional–shearing vibrations by using the principle of virtual displacements. Kim and Kim [9] proposed an improved shear deformable thin walled beam theory by introducing Vlasov's assumption and applying Hellinger–Reissner principle which includes the shear deformations due to the shear forces and the restrained warping torsion and due to the coupled effects between them. Some valuable papers about thin walled curved beams should be mentioned as Yoon et al. [10] and Piovan and Cortínez [11], among others, who investigated the free or forced vibrations of thin-walled curved beams. Regularly, new theories and approaches are presented in the literature and many comparisons are performed between them. However, in most cases, it is notable the lack of experimental data that it is essential to compare with numerical results in order to obtain reliable conclusions about the accuracy and applicability of different theories. Ambrosini [12] presented a thin walled beam theory and its verification with experimental tests.

In what concerns to beams axially loaded, Hashemi and Richard [13] presented a dynamic finite element for the natural frequencies and modes calculation of coupled bending–torsional vibration of axially loaded beams based on the closed form

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solutions of the Bernoulli–Euler and St. Venant beam theories. Li et al. [14] extended the development of Arpaci and Bozdag [15] and presents a dynamic transfer matrix method of determining the natural frequencies and mode shapes of axially loaded thin-walled Timoshenko beams. Kim et al. [16] proposed an improved numerical method to exactly evaluate the dynamic and static element stiffness matrices for the spatial free vibration and stability analysis of non-symmetrical thin-walled straight beams subjected to eccentrically axial loads. The equations of motion are derived from the total potential energy. Chen and Hsiao [17] analyzed the coupled axial–torsional vibration of thin-walled *Z*-section beam induced by the boundary conditions. The natural frequencies are obtained using the bisection method. Viola et al. [18] investigated the changes in the magnitude of natural frequencies and modal response introduced by the presence of a crack on an axially loaded uniform Timoshenko beam. Kim et al. [19] presented the static and dynamic stiffness matrices for the flexural–torsional buckling and free vibration analysis of thin-walled beam with non-symmetric cross-section subjected to linearly variable axial force based on the power series method. To the best knowledge of the authors, there are no experimental results for thin walled beams axially loaded published in the specialized literature.

In this paper, the equations of transversal vibration of thin walled beams with a constant axial load are presented both in the time and frequency domains using the state variables approach. A numerical study based on the equations developed is presented and the influence of a tension and compression axial load in free vibrations is discussed.

Finally, results of experimental tests are shown because it is the way in which the results obtained by different theories should be verified. These experimental results are used to validate the equations presented in this paper and can be used for checking the accuracy and reliability of calculation methods and procedures.

2. Theory

2.1. Equations of motion

Following Vlasov's convention, the left-handed rectangular global coordinates system (*x, y, z*) shown in Fig. 1 was adopted. The associated displacements are designated ξ, η and ζ. In the figure, A represents the centroid and O the shear center.

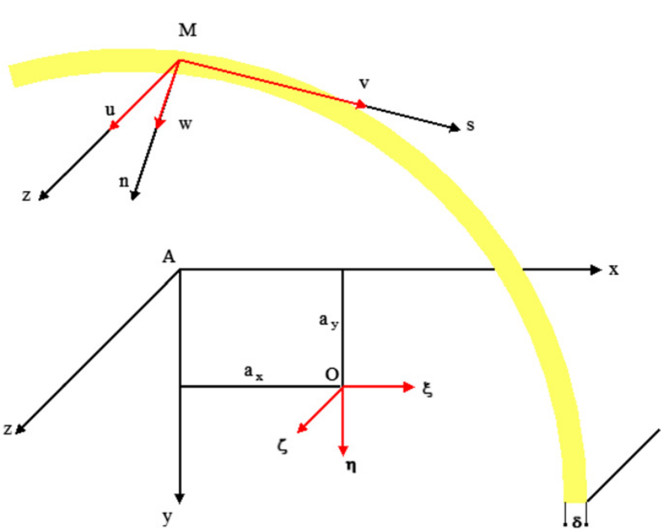

Fig. 1. Coordinate systems and associated displacements.

A constant axial eccentric force P was introduced to the physical model developed by Ambrosini et al. [4,5]. For the case of free vibrations and uniform section, the model in the time domain is given by the following three fourth-order partial differential equations in the generalized displacements ξ, η and θ:

$$EJ_y\left(\frac{\partial^4\xi}{\partial z^4}-\frac{\partial^3\gamma_{mx}}{\partial z^3}\right)-P\left(\frac{\partial^2\xi}{\partial z^2}-\frac{\partial\gamma_{mx}}{\partial z}\right)+P(e_y-a_y)\frac{\partial^2\theta}{\partial z^2}+\rho F_T\left(\frac{\partial^2\xi}{\partial t^2}+a_y\frac{\partial^2\theta}{\partial t^2}\right)-\rho J_y\left(\frac{\partial^4\xi}{\partial z^2\partial t^2}-\frac{\partial^3\gamma_{mx}}{\partial z\partial t^2}\right)=0, \tag{1a}$$

$$EJ_x\left(\frac{\partial^4\eta}{\partial z^4}-\frac{\partial^3\gamma_{my}}{\partial z^3}\right)-P\left(\frac{\partial^2\eta}{\partial z^2}-\frac{\partial\gamma_{my}}{\partial z}\right)-P(e_x-a_x)\frac{\partial^2\theta}{\partial z^2}+\rho F_T\left(\frac{\partial^2\eta}{\partial t^2}-a_x\frac{\partial^2\theta}{\partial t^2}\right)-\rho J_x\left(\frac{\partial^4\eta}{\partial z^2\partial t^2}-\frac{\partial^3\gamma_{my}}{\partial z\partial t^2}\right)=0, \tag{1b}$$

$$EJ_\phi\frac{\partial^4\theta}{\partial z^4}-GJ_d\frac{\partial^2\theta}{\partial z^2}+P\left(\frac{\partial^2\xi}{\partial z^2}-\frac{\partial\gamma_{mx}}{\partial z}\right)(e_y-a_y)-P\left(\frac{\partial^2\eta}{\partial z^2}-\frac{\partial\gamma_{my}}{\partial z}\right)(e_x-a_x)-P(r^2+2e_x\beta_x+2e_y\beta_y)\frac{\partial^2\theta}{\partial z^2}++\rho F_T\left(a_y\frac{\partial^2\xi}{\partial t^2}-a_x\frac{\partial^2\eta}{\partial t^2}+r^2\frac{\partial^2\theta}{\partial t^2}\right)-\rho J_\phi\frac{\partial^4\theta}{\partial z^2\partial t^2}=0. \tag{1c}$$

In these equations, F_T is the cross-sectional area, J_x and J_y are the moments of inertia of the cross-section about the principal axes, J_φ the sectorial moment of inertia, J_d the torsion modulus, a_x and a_y the coordinates of the shear center, and e_x and e_y the eccentricity of the axial load *P*. ρ denotes the mass density of the beam material, *E* and *G* are the Young's and the shear modulus, respectively, and γ_{mx} and γ_{my} represent the mean values of shear strains over a cross-section z=constant.

$$\gamma_{mx}=\frac{Q_x}{k_xF_TG}, \tag{2a}$$

$$\gamma_{my}=\frac{Q_y}{k_yF_TG}. \tag{2b}$$

where Q_x and Q_y are shear stresses on the cross-section and k_x and k_y denote the Cowper's shear coefficients.

r, β_x and β_y are geometric characteristics:

$$r^2=a_x^2+a_y^2+\frac{J_x+J_y}{F_T}, \tag{3}$$

$$\beta_x=\frac{U_y}{2J_y}-a_x, \tag{4a}$$

$$\beta_y=\frac{U_x}{2J_x}-a_y. \tag{4b}$$

$$U_x=\int_F y^3dF+\int_F x^2ydF, \tag{5a}$$

$$U_y=\int_F x^3dF+\int_F y^2xdF. \tag{5b}$$

The right terms of expression (5) are third-order moments and products of inertia of the considered section.

The system (1a)–(1c) represents a general model of beams that take into account triply coupled flexural–torsional vibrations. It must be pointed out that the longitudinal vibration equation related to the generalized displacement ζ (Fig. 1) is non-coupled