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Generalised capacity curves for stability and plasticity: Application and limitations

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ABSTRACT

In recent decades, the resistance of a structure has been thought of as well defined by the outcome of a geometrically and materially nonlinear analysis with explicitly modelled imperfections (GMNIA). But when this is the only analysis that is performed on a complex structural system, it is sometimes difficult to interpret the result. The outcome must be seen in the context of those from simpler analyses, which can define appropriate reference quantities.

Other analyses, like a small displacement theory materially nonlinear analysis (MNA) and a linear elastic bifurcation analysis (LBA) are very important in the interpretation of a GMNIA.

The general capacity curve in the Eurocode for shell structures [1] provides a representation of these different analyses. Using this capacity curve, different identifiable key aspects of the structure's behaviour can be studied independently and understood in relation to the corresponding parameter of this curve. This unified representation allows an easy and meaningful characterisation of all elastic–plastic buckling problems.

However, some care is needed when applying such a generalised curve to structures with particular features. This paper outlines the limitations of the simplest version of the curve, and develops an enhancement that permits it to be deployed without restriction.

1. Introduction

The capacity curve, presenting the interaction between plasticity and stability in a structural mechanics problem, is a powerful tool for both design and research. It describes the practical compressive resistance of a structural system or component in terms of its slenderness. Such curves, often referred to as a "column curve" though applied to other structures, are normally empirically devised for each structure and load case separately. However, the recent development of a generalised capacity curve [1] that can be used for a wide variety of different structures has opened new opportunities. This curve is expressed in terms of a small number of key parameters, each of which relates to a specific aspect of the structural behaviour, so these parameters can be used to gain deeper insights into the controlling phenomena for the structure [2].

This paper describes the background to the generalised capacity curve adopted into EN 1993-1-6 [1], and shows its usefulness in application to several shell structures. However, some care is needed when applying such a generalised curve to structures with particular and perhaps unusual features, such as yielding at a very low proportion of the plastic failure resistance or structural systems in which yielding and buckling occur in very different locations. This paper outlines the limitations of the simplest version of the curve, and develops an enhancement that permits it to be deployed without restriction.

It is hoped that this generalised capacity curve will be widely adopted for studies of all structural systems and forms, so that the strong reliance on special case empiricism that currently dominates this part of structural design can be reduced, and new insights can be gained into the structural behaviour of complex systems.

The capacity curve describes the behaviour of a structural system in terms of two key resistances, as defined by EN 1993-1-6 [1]. The first of these is the lowest elastic linear eigenvalue, derived from a linear elastic small displacement theory bifurcation analysis (LBA). The second is a small displacement theory plastic limit analysis using ideal elastic–plastic material properties, which identifies the plastic collapse load of the system (MNA). These two loads are the only two limiting loads that can always be determined for any structural system, since all other analyses may show indefinite hardening behaviour and be difficult to interpret [3].

For complex systems it is very difficult to be certain of calculating the true maximum load using a fully geometrically and materially

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nonlinear analysis with explicit imperfections (GMNIA) since the identification of appropriate imperfection forms and amplitudes can lead to a very tedious search. So it cannot be expected that GMNIA will be used in design, and the drafting panel for EN 1993-1-6 [1] devised instead a methodology termed MNA-LBA in which only the two resistances defined above would be calculated numerically and the remaining phenomena deduced from the expected form of the capacity curve. This method places special importance on the parameters of the capacity curve. In structural framework design, this method has been adopted, with modifications, into [4] and termed the "General Method", but it too relies on the parameters of the capacity curve being known a priori. These developments make the general capacity curve described in this paper of even more importance since some of the parameters within it could potentially be deduced from simpler analyses than a full GMNIA. A fuller description of this methodology is given in [5].

2. The traditional capacity curve

The concept behind the capacity curve was probably first proposed by Rankine [6] when he identified the two limits of column strength as governed by material failure and by elastic buckling, with his interaction between them providing the first such curve. Later researchers [7–12] expanded on the concept, developing different theoretical backgrounds that continue to be used up to the present day. Comparable curves have been used for beams, plates and shells in the many years since these ideas were first proposed, but only limited and rather crude attempts have been to apply them to structural systems [13], until the development described here.

One unfortunate historical feature of the column curve that is the original basis of this interaction between plasticity and stability is the struggle that occurred between the proponents of different theoretical bases in the 1950s. The followers of Ayrton and Perry [7] adopted the concept of an imperfect column, but had to use first yield as a criterion of failure. By contrast the followers of Considère [8] and Engesser [9] believed in a perfect column and used tangent modulus theory to determine when bifurcation would occur. The critics of the former pointed out that structures yield a lot before failure so it had a poor basis in mechanics, whilst the critics of the latter pointed out that all structures were imperfect so this inelastic theory idealised the structure excessively. The adoption of a Perry treatment in the EN 1993-1-1 [4] unfortunately merely adopts one of these positions and does not move the debate onwards. The curve of this paper attempts to avoid such sterile arguments.

The essential feature of these capacity curves is that two limiting cases are clearly defined: one based on pure material failure, the other based on elastic stability failure, and an interposing relation between the two is defined, using the "slenderness" of the structure as the measure between them. The general expression for slenderness λ is derived from columns as

$$\lambda = \sqrt{\frac{R_{pl}}{R_{cr}}} \tag{1}$$

where R_{pl} is the resistance to material failure (plasticity) and R_{cr} is the elastic critical resistance. For a structural system subject to a complex load set, it is not possible to define these resistances in terms of a force or bending moment, torque or bimoment or stress, since none of these can capture the complete state of the structure. Instead, the design load condition of the structure must be defined, and analyses can be performed to see how much those loads can be increased before failure in a particular mode. The resulting load incrementation factors are the structure's resistance in that mode to that load set, and are consequently defined as R in Rotter and Schmidt [5].

3. The new generalised capacity curve

The recent development of a generalised capacity curve [1] that can be used for a wide variety of different structures and structural systems has opened new opportunities. As noted in the introduction, this curve is expressed in terms of a small number of key parameters, each of which relates to a specific aspect of the structural behaviour, so these parameters can be used to gain deeper insights into the controlling phenomena for the structure [2]. These key parameters represent the major advantage of the generalised capacity curve over the earlier empirical column curves.

This capacity curve, proposed by [14], describes the complete behaviour of a structure from a fully plastic collapse at a low slenderness, $\lambda = \lambda_0$, through plastic buckling, $\lambda_0 < \lambda < \lambda_p$, to elastic buckling including imperfections at high slenderness ($\lambda > \lambda_p$) using the relative slenderness λ of the structure (Fig. 1) and very few parameters to define the failure behaviour of the structure.

The relative slenderness of the structural system λ is still defined by Eq. (1), with R_{pl} as the plastic limit resistance derived from (R_{MNA}) from a small displacement theory ideal elastic–plastic analysis (MNA) and R_{cr} is the elastic critical resistance (R_{LBA}) from a linear bifurcation analysis (LBA).

The dimensionless resistance parameter χ is defined as

$$\chi = R_f / R_{pl} \tag{2}$$

where the dimensionless resistance R_f is the failure load factor, found in an experiment or calculated using a geometrically and materially nonlinear analysis of the imperfect structure (R_{GMNIA}).

The shape of the capacity curve of EN 1993-1-6 [1] is given by

$$\chi = 1 \quad \text{when} \quad \bar{\lambda} \le \bar{\lambda}_0 \tag{3}$$

$$\chi = 1 - \beta \left(\frac{\bar{\lambda} - \bar{\lambda}_0}{\bar{\lambda}_p - \bar{\lambda}_0} \right)^{\eta} \quad \text{when} \quad \bar{\lambda}_0 < \bar{\lambda} < \bar{\lambda}_p \tag{4}$$

$$\chi = \alpha / \bar{\lambda}^2 \quad \text{when} \quad \bar{\lambda}_p < \bar{\lambda} \tag{5}$$

with

$$\bar{\lambda}_p = \sqrt{\alpha/(1-\beta)} \tag{6}$$

Eqs. (3)–(6) were originally proposed by [14]. Eq. (3) was later modified [3] to include a hardening zone as

$$\chi = \chi_h - (\bar{\lambda}/\bar{\lambda}_0)[\chi_h - 1] \quad \text{when} \quad \bar{\lambda} \le \bar{\lambda}_0 \tag{7}$$

The capacity curve has three sections: in the first (Eq. (3) or (7)) the resistance is equal to or exceeds the plastic limit resistance R_{pl} (derived from an MNA analysis) for slendernesses smaller than the squash limit relative slenderness λ_0.

Resistance above R_{pl} are found in all numerical analyses if any one of the following three features is present: (a) strain hardening is included, (b) changes in geometry (geometric hardening) raises

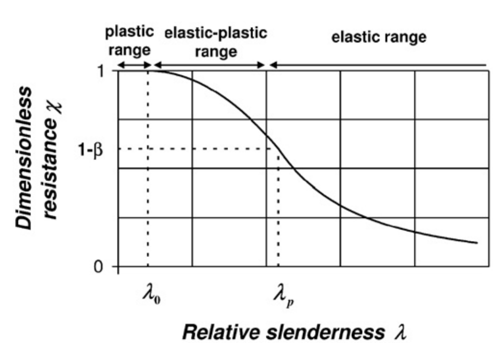

Fig. 1. Generalised capacity curve.