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A beam model for large displacement analysis of flexibly connected thin-walled beam-type structures

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ABSTRACT

This paper presents a beam formulation for large displacement analysis of beam-type structures with flexible connections. Within the framework of updated Lagrangian incremental formulation and the nonlinear displacement field of thin-walled cross-sections, which accounts for restrained warping and the second-order displacement terms due to large rotations, the equilibrium equations of a straight beam element are firstly developed. Due to the nonlinear displacement field, the geometric potential of semitangential moment is obtained for both the internal torsion and bending moments, respectively. Material nonlinearity is introduced for an elastic-perfectly plastic material through the plastic hinge formation at finite element ends. To account for the flexible connection behaviour, a special transformation procedure is developed. The numerical algorithm is implemented in a computer programme and its reliability is validated trough several test examples.

1. Introduction

In the large displacement or second-order analysis of thin-walled beam-type structures, two extreme idealisations for connections are frequently used: fully rigid and frictionless pinned [1–5]. Such models simplify the large displacement analysis significantly, but often cannot represent the real structural behaviour because real connections exhibit a flexible behaviour which falls in between the two idealised cases. Flexibility of connections is the result of a complex interaction among various components of the connection construction itself [6–13]. Therefore, conventional numerical analysis procedures must be broadened by incorporating real connection characteristics in order to replace the idealised connection approach, which improves the accuracy of structural analysis [14–20].

To perform the large displacement analysis for a beam-type structure, a nonlinear beam model should be made available, with which the load–displacement behaviour of a frame structure can be obtained by one of the incremental description [21–24]. Each description utilises a different structural configuration for describing the system quantities, based on which a set of nonlinear equilibrium equations can be derived for the structure. This set can be further linearised and solved using some incremental-iterative scheme, which consists of three main phases. The first or predictor phase comprises evaluating the overall structural stiffness and solving for the displacement increments from the approximated incremental equilibrium equations for the structure. Using the standard transformation process, displacement increments of each finite element can be determined immediately. The second or corrector phase involves the updating of nodal coordinates as well as orientations of cross-sections and axes of each element, and the determination of the exact nodal forces for each element using a particular force recovery algorithm. The third or checking phase is to check if the convergence criterion of iteration adopted is achieved in the current configuration by comparing with the preset tolerance value.

This paper presents an elastic–plastic beam element for the large displacement analysis of beam-type structures composed of the straight and prismatic thin-walled beam members and flexible joints. The geometric nonlinearity is first analysed for the case of linearly elastic material behaviour. In this, it is assumed that the cross-section is not deformed in its own plane, but is subjected to warping in the longitudinal direction. Shear strains in the middle surface are neglected. Displacements are allowed to be large, but strains are assumed to stay small. External loads are static and conservative. Internal moments are represented as the stress resultants calculated by the Euler–Bernoulli–Navier theory for bending and the Timoshenko–Vlasov theory for torsion [25,26]. The element geometric stiffness is derived using the updated Lagrangian (UL) description and the nonlinear displacement field of a thin-walled cross-section, which includes the second-order displacement terms due to large rotation effects [27,28]. In such

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a way, the incremental geometric potential corresponding to the semitangential moment [29–31] is obtained for the internal torsion and bending moments, respectively, thereby ensuring the moment equilibrium conditions to be preserved at the joint to which beam members of different orientations are connected. The generalised displacement control method is employed as an incremental-iterative solution scheme [32]. At the end of the each iteration, the updating of nodal orientations is performed using the transformation rule which applies for semitangential incremental rotations [33], while the force recovering is performed according to the conventional approach (CA) [34].

Elastic–plastic behaviour is introduced by the plastic hinge concept [35,36], in the sense that plastic deformations are assumed to be confined to zero-length plastic hinges at the beam element ends, while the material is assumed to be elastic-perfectly plastic with no strain hardening. Supposing the existence of a continuous and convex single-function yielding surface in terms of the beam-stress resultants obtained by the adopted force recovery algorithm, a plastic reduction matrix is introduced into the incremental equilibrium equations of the beam element [37,38].

A hybrid element, hereafter called the SR element, composed of the aforementioned nonlinear beam element and dimensionless linear/nonlinear springs added at element nodes is introduced for modelling the structures at which flexible connections may occur. One side of each spring is connected to a node of the beam element, while the other side is connected to a global node. Using the SR beam element, connections are no longer assumed to be fully rigid.

2. Basic considerations for thin-walled beam

2.1. Kinematics of beam

The deformation of an initially straight prismatic beam with a thin-walled cross-section of a wall of thickness t is considered. For the sake of simplicity, it is assumed that the shear centre and the centroid of the cross-section coincide. A right-handed Cartesian coordinate system (z, x, y) is chosen in such a way that z-axis coincides with the beam axis passing through the centroid O of each cross-section, while the x- and y-axes are the principal inertial axes of the cross-section. Incremental displacement measures of a cross-section are defined as

$$w_o = w_o(z), \quad u_o = u_o(z), \quad v_o = v_o(z), \quad \varphi_z = \varphi_z(z),$$
$$\varphi_x = -v_o' = \varphi_x(z), \quad \varphi_y = u_o' = \varphi_y(z), \quad \theta = -\varphi_z' = \theta(z) \tag{1}$$

where w_o, u_o and v_o are the rigid-body translations of the cross-section associated with the centroid in the z-, x- and y-directions, respectively; φ_z, φ_x and φ_y are the rigid-body rotations about the z-, x- and y-axis, respectively; θ is a parameter defining the warping of the cross-section. The superscript 'prime' indicates the derivative with respect to z.

If rotations are small, the incremental displacement field of a thin-walled cross-section contains only the first-order displacement terms [39]:

$$u_z = w = w_o - y\ v_o' - x\ u_o' - \omega\ \varphi_z';$$
$$u_x = u = u_o - y\ \varphi_z, \quad u_y = v = v_o + x\ \varphi_z \tag{2}$$

in which w, u and v are the linear or first-order displacement increments of an arbitrary point on the cross-section defined by the position coordinates x and y and the warping function $\omega(x, y)$. If the assumption of small rotations is not invalid, then the second-order displacement increments [28]:

$$\tilde{u}_z = \tilde{w} = 0.5(x\varphi_z\varphi_x + y\varphi_z\varphi_y);$$
$$\tilde{u}_x = \tilde{u} = 0.5[-x(\varphi_z^2+\varphi_y^2) + y\varphi_x\varphi_y]$$
$$\tilde{u}_y = \tilde{v} = 0.5[x\,\varphi_x\,\varphi_y - y(\varphi_z^2+\varphi_x^2)] \tag{3}$$

due to large rotations should be added to those from Eq. (2). The corresponding incremental Green–Lagrange strain tensor can be written as

$$\varepsilon_{ij} \cong e_{ij} + \eta_{ij} + \tilde{e}_{ij}; \quad e_{ij} = 0.5(u_{i,j} + u_{j,i});$$
$$\eta_{ij} = 0.5u_{k,i}\,u_{k,j}; \quad \tilde{e}_{ij} = 0.5(\tilde{u}_{i,j} + \tilde{u}_{j,i}) \tag{4}$$

in which the last strain term is due to large rotations. According to the geometrical hypothesis of in-plane rigidity of the cross-section, the strain components $\varepsilon_{11}=\varepsilon_{xx}$, $\varepsilon_{22}=\varepsilon_{yy}$ and $2\varepsilon_{12}=2\varepsilon_{xy}=\gamma_{xy}$ in Eq. (4) should be equal to zero.

2.2. Stress resultants

In the conventional engineering theories for bending and torsion, the stress components $\sigma_x=\sigma_y=\tau_{xy}=0$ are assumed to vanish, and the stress resultants acting on each cross-section can be defined as follows [25,26]:

$$F_z = \int_A \sigma_z\,\mathrm{d}A; \quad F_x = \int_A \tau_{zx}\,\mathrm{d}A; \quad F_y = \int_A \tau_{zy}\,\mathrm{d}A$$
$$M_z = \int_A (\tau_{zy}x - \tau_{zx}y)\,\mathrm{d}A = T_{SV} + T_\omega; \quad M_x = \int_A \sigma_z y\,\mathrm{d}A$$
$$M_y = -\int_A \sigma_z x\,\mathrm{d}A\,; \quad M_\omega = \int_A \sigma_z\,\omega\,\mathrm{d}A; \quad \overline{K} = \int_A \sigma_z(x^2+y^2)\,\mathrm{d}A \tag{5}$$

where F_z represents the axial force, F_x and F_y are the shear forces, M_z is the torsion moment, M_x and M_y are the bending moments with respect to the x- and y-axis, respectively, M_ω is the bimoment, while $\overline{K}$ is the Wagner coefficient. As can be seen from the preceding equation, the torsion moment consists of two parts T_{SV} and T_ω representing the St. Venant or uniform torsion moment and the warping or non-uniform torsion moment, respectively.

By assuming the stress–strain relations in the linearised incremental sense as $\sigma_z=E\ e_{33}=E\ e_z$, $\tau_{zx}=2G\ e_{31}=G\ e_{zx}$ and $\tau_{zy}=2G\ e_{32}=G\ e_{zy}$, where E and G are the elastic and shear moduli, Eq. (5) can be manipulated to yield:

$$F_z = EA\frac{\mathrm{d}w_o}{\mathrm{d}z}; \quad M_x = -EI_x\frac{\mathrm{d}^2v_o}{\mathrm{d}z^2}, \quad M_y = EI_y\frac{\mathrm{d}^2u_o}{\mathrm{d}z^2}; \quad T_{SV} = GI_t\frac{\mathrm{d}\varphi_z}{\mathrm{d}z}$$
$$M_\omega = -EI_\omega\frac{\mathrm{d}^2\varphi_z}{\mathrm{d}z_z^2}; \quad \overline{K} = F_z\alpha_z + M_x\alpha_x + M_y\alpha_y + M_\omega\alpha_\omega. \tag{6}$$

Eq. (6) represents the linearised incremental force–displacement relationships, in which A is the cross-sectional area, I_x and I_y are the principal moments of inertia about x- and y-axis, respectively, I_t is the St. Venant torsion constant, I_ω is the warping moment of inertia, while the coefficients α_z, α_x, α_y and α_ω are the corresponding cross-section parameters provided in Ref. [39]. The shear forces as well as the non-uniform torsion moment are treated as reactive forces and can be determined as $F_x=-\mathrm{d}M_y/\mathrm{d}z$, $F_y=\mathrm{d}M_x/\mathrm{d}z$ and $T_\omega=\mathrm{d}M_\omega/\mathrm{d}z$.

3. Finite element formulation

3.1. Incremental description

According to the incremental description, it is necessary to subdivide a load–deformation path of a finite element into a number of steps or increments where three equilibrium configurations can be recognised: the initial configuration C_0, the last calculated equilibrium configuration C_1 and current unknown configuration C_2. By the UL formulation adopted in this paper, each system quantity occurring in C_2 can be expressed with reference to C_1. Hereafter, a left superscript denotes the configuration in which a quantity occurs, and a left subscript the configuration in which the quantity is measured. If the superscript and subscript of a quantity are same, the latter may be