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Buckling and free vibration analysis of stiffened panels

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ABSTRACT

The paper presents the application of the new stiffener element with seven degrees of freedom per node and the subsequent application in determining frequencies, mode shapes and buckling loads of different stiffened panels. In structural modelling, the stiffener and the plate/shell are treated as separate elements where the displacement compatibility transformation between the seven and six degrees of freedom nodes of these two types of elements takes into account the torsion–flexural coupling in the stiffener and the eccentricity of internal (contact) forces between the beam–plate/shell parts. The model becomes considerably more flexible due to this coupling technique. The development of the stiffener is based on a general beam theory, which includes the constraint torsional warping effect and the second-order terms of finite rotations. Numerical tests are presented to demonstrate the importance of torsion warping. As part of the validation of the results, complete shell and the usual six degrees of freedom per node shell–beam finite element analyses were made for stiffened panels.

1. Introduction

Many engineering structures consist of stiffened thin plate and shell elements to improve the strength to weight ratio. The buckling and vibration characteristics of stiffened plates and shells are of considerable importance to mechanical and structural engineers.

Among the known solution techniques, the finite element method is certainly the most favourable. Using the technique where stiffeners are modelled by beam finite elements, Jirousek [1] formulated a four-node isoparametric beam element including transverse shear and Saint-Venant torsion effects. More recent studies on dynamic and buckling problems of stiffened plates and shells are available in Refs. [2–5]. It is a common feature of these finite element-based methods that in order to attain displacement continuity, a rigid fictitious link is applied to connect one node in the plate element to the beam node shearing the same section. This approach neglects the out-of-plane warping displacements of the beam section and, in such cases, the usual formulation overestimates the stiffener torsional rigidity. To eliminate this problem Patel et al. in Ref. [5] introduced a torsion correction factor which is analogous to the shear correction factor commonly used in the shear deformation beam theory.

The main objects of the present paper is to propose an efficient procedure modelling the connection between the plate/shell and the stiffener, and as part of it the constraint torsion effect in the stiffener. Apart from Refs. [6,7], where the so-called two interface line concept was used, the author could not find any work in the literature involved in the examination of constrained torsion in the stiffener of complex plate/shell structures. However, the effect is obvious, especially in terms of dynamic and stability phenomena when the global characteristics of a structure are investigated, such as frequencies, mode shapes, or critical loads causing a loss of stability. Investigations of stand-alone beam structures proved that an approximate or more accurate modelling of the torsional stiffness, eccentricity, or mass distribution can considerably modify the results. Theoretically—and practically as well, if there is adequate capacity available—beam-type components in complex structures can also be modelled by flat shell, or even spatial finite elements. Consequently, the size of the model and the number of degrees of freedom (dof) will change considerably, increasing the time required for calculations and making the interpretation and evaluation of results more difficult. It is a better solution if the properties of components are improved and the ranges of phenomena possible to be modelled are increased at the element level.

As the main objective of this paper is to study the effect of constraint torsion and the coupling methods, the shear deformation of the beam is neglected and the formulation of the stiffener is based on the well-known Bernoulli–Vlasov theory. For the finite element analysis, cubic Hermitian polynomials are utilized as the beam shape functions of lateral and torsional displacements. The stiffener element has two nodes with seven dof per node. In order to maintain displacement compatibility of the seven and six dof nodes of the beam and the stiffened element, a special transformation is used, which includes the coupling of torsional

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Nomenclature

A	cross-section area of stiffener
B	bimoment
E	Young's modulus
F_x, F_y, F_z	initial external forces
G	shear modulus
I_r, I_s	principal moments of inertia
I_ω	warping constant of stiffener
J	St. Venant's torsion constant
L	beam element length
M_r, M_s	bending moments about r, s axes
M_t	torsional moment
M_1, M_2, M_3	internal moments with respect to shear centre axes
M_W	Wagner's moment
N_j, N_1, N_2	nodes of beam element
N	axial force
r, s	principal axes of stiffener section
S	shear centre of stiffener section
$\bar{u}$	average of axial displacement
u, v, w	displacement increments of the shear centre S
V_r, Vs	shear forces
y_{NC}, z_{NC}	node–centriod eccentricity
y_{CS}, z_{CS}	centriod–shear centre eccentricity
y_{SP}, z_{SP}	load eccentricity from shear centre
$\mathbf{k}_{Ge}$	element external load stiffness matrix
$\mathbf{k}_{Gi}$	element geometric stiffness matrix
$\mathbf{k}_L$	element linear stiffness matrix
$\mathbf{m}$	element mass matrix
W	work done by load increments
α, β, γ	rotation increments of the shear centre S
β_r, β_s, β_ω	Wagner's mono-symmetry properties
δ	stiffener-plate area ratio parameter
φ	torsional warping function
ϑ	warping parameter (rate of twist)
π_G	energy due to initial stress resultants
π_{Ge}	energy due to initial external loads
π_L	elastic strain energy
ρ	mass density
λ	buckling load factor
ω	natural frequency of vibration

and bending rotations and the eccentricity of internal forces between the stiffener and the plate elements.

2. Beam element

2.1. Kinematics of beam

In this work, the basic assumptions are as follows: the beam member is straight and prismatic, the cross-section is not deformed in its own plane but is subjected to torsional warping, rotations are large but strains are small, the material is homogeneous, isotropic and linearly elastic.

Let us have a straight, prismatic beam member with an arbitrary cross-section as it is shown in Fig. 1. The local x-axis of the right-handed orthogonal system is parallel to the axis of the beam and passes through the end nodes N_1 and N_2. The co-ordinate axes y and z are parallel to the principal axes, marked as r and s. The positions of the centroid C and shear centre S in the plane of each section are given by the relative co-ordinates y_{NC}, z_{NC} and y_{CS}, z_{CS}. The external loads are applied in points P located y_{SP} and z_{SP} from the shear centre.

Based on semitangential rotations, the displacement (specifically, the incremental displacement) vector consisting of translational, rotational and warping components is obtained as

$$\mathbf{u} = \mathbf{U} + \mathbf{U}^*, \tag{1}$$

where $\mathbf{U}$ and $\mathbf{U}^*$ are the displacements corresponding to the linear part and second-order terms due to large rotations:

$$\mathbf{U} = \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix} = \begin{bmatrix} u + \vartheta\varphi \\ v \\ w \end{bmatrix} + \begin{bmatrix} \beta(s - z_{CS}) - \gamma(r - y_{CS}) \\ -\alpha(s - z_{CS}) \\ \alpha(r - y_{CS}) \end{bmatrix}, \tag{2}$$

$$\mathbf{U}^* = \begin{bmatrix} U_x^* \\ U_y^* \\ U_z^* \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \alpha\beta(r - y_{CS}) + \alpha\gamma(s - z_{CS}) \\ -(\alpha^2 + \gamma^2)(r - y_{CS}) + \beta\gamma(s - z_{CS}) \\ \beta\gamma(r - y_{CS}) - (\alpha^2 + \beta^2)(s - z_{CS}) \end{bmatrix}. \tag{3}$$

Displacement parameters are defined at the shear centre S as shown in Fig. 2. Accordingly, u, v and w are the translations of point S and α, β and γ denote rigid body rotations about the shear centre axes parallel to x, y and z, respectively. The small out-of-plane torsional warping displacement is defined by the $\vartheta(x)$ warping parameter and the $\varphi(r,s)$ warping function normalized with respect to the shear centre. In the following, the warping function φ and the shear centre location are the same as in the case of free torsion. For thin-walled sections $\varphi = -\omega$, the sector area co-ordinate. When the shear deformation effects are not considered, the Euler-Bernoulli and the Vlasov internal kinematical constraints are adopted as

$$\beta = -w', \quad \gamma = v', \quad \vartheta = \alpha', \tag{4}$$

where the prime denotes differentiation with respect to variable x.

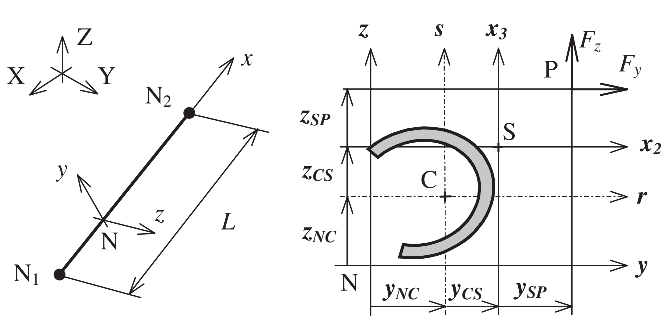

Fig. 1. Beam element local systems and eccentricities.

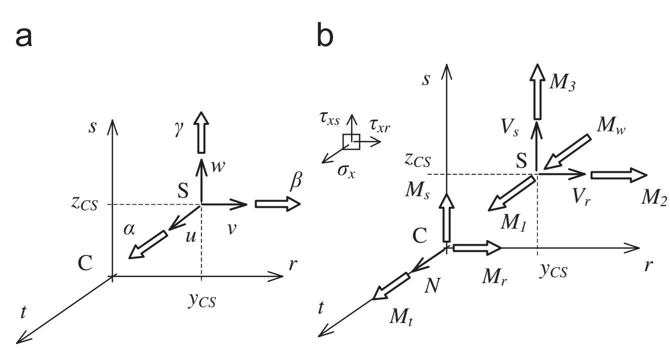

Fig. 2. Notation of displacement parameters and stress resultants.