Thin-Walled Structures 48 (2010) 215–222

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Meshfree analysis of dynamic fracture in thin-walled structures

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ARTICLE INFO

Article history:
Received 11 September 2008
Received in revised form
13 October 2009
Accepted 26 October 2009
Available online 26 November 2009

Keywords:
Cylindrical shell
Fracture
Crack
Dynamics
EFG
Thin-walled structure

ABSTRACT

Analysis and reliability assessment of fracturing thin-walled structures is important in engineering science. We focus on numerical analysis of dynamic fracture of thin-walled structures such as pipes and pressure vessels. Instead of using finite element method, we propose meshfree method that has advantages because its higher order continuity and smoothness and its opportunities to model fracture in a simple way. Therefore, connectivity between adjacent nodes are simply removed once fracture criterion is met. The main advantage of our meshfree method is its simplicity and robustness.

1. Introduction

The numerical analysis of fracture in thin-walled structures is still a challenge in applied mechanics that poses essential difficulties on the numerical method used [1,10–15,18,22,26,27,31,35–37,40,42,43,46,47]. Thin-walled structures are used in many components such as pipes, vessels or sheets. Fracture of such structure can have different causes, ranging from static loading to internal pressure and gas explosions or impacts. Modeling of thin-walled structures is important in many engineering applications, e.g. the analysis of sheet metal forming, crash-worthiness test, civil structure design, pressure vessel liability, shipbuilding, defense technology, just to name a few. Especially the modeling of fracture in thin-walled structures imposes severe difficulties on existing numerical methods. Numerical analysis of thin-walled cylindrical structures can be classified into three categories:

(1) Numerical analysis based on shell theories.
(2) Degenerated continuum, or continuum based approach.
(3) Direct three-dimensional (3-D) continuum approach.

Among these three approaches, 3-D continuum direct approach is the simplest. The first two approaches lead to complicated theories and the development of particular constitutive models. 3-D continuum models cannot easily be adopted. When the 3-D continuum approach is used in the finite element framework, the method is cumbersome since multiple elements need to be used over the thickness of the thin shell to acquire reasonable gradient fields. This makes the method computationally expensive.

Therefore, we use meshfree method instead of finite element method. Meshfree approximations have several advantages for modeling thin-walled structures. The two main advantages are:

- Their shape functions are smooth and higher order continuous and therefore more accurate than finite element methods. Only a layer of 2–3 nodes over the thickness are required that makes the method computationally efficient. More importantly, the critical time step in meshfree methods based on nodal integration is significantly larger than for three-dimensional finite elements. Ref. [29] have shown that the critical time step is more than 100 times larger. We based the time step on nodal distances in circumferential direction and did not observe any instabilities with such a time step. Neither did we observe an influence when we decreased and increased the chosen time step by a factor of 2–3.
- Fracture can be incorporated in simple manner, i.e. by removing connectivities between meshfree nodes. Fracture in finite elements can be only realized by computational expensive remeshing procedures.

Most meshfree methods proposed so far have focused on a continuum-based approach. A meshfree thin shell formulation based on Kirchhoff–Love theory and element-free Galerkin (EFG) method [3] has been developed by [17] in the context of small strain, linear elastic framework. Ref. [41] extended this work with the consideration of finite strain, non-linear elastic material and

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doi:10.1016/j.tws.2009.10.011

focused on fracture. In the following work, [30] have simplified the treatment of cracks in thin shell by using an extrinsic basis. Ref. [8] noted the advantage of meshfree approximations in addressing shear locking in Mindlin type of beams and plates and have developed a meshfree formulation based on the reproducing kernel particle method (RKPM) [21]. This methodology is further extended by [16] with the use of EFG. EFG has been employed by [25] for shell and membrane structures in which bi-cubic and quartic basis functions are introduced in order to avoid shear and membrane locking. Ref. [48] showed that the Kirchhoff mode in the Mindlin plate can be reproduced using EFG or RKPM if second-order polynomial basis is used in the moving least-squares approximation. By implementing this with a nodal integration and stabilization scheme, they have shown that the formulation is stable and free of shear locking. Ref. [49] developed a free mesh method in which the discrete Kirchhoff theory is combined with the mixed approach. In the case of three-dimensional continuum models, [20] have presented a formulation based on RKPM and have studied non-linear large deformation of thin shells.

In this paper, we use three-dimensional meshfree continuum approach. Fracture of the shell is modeled by breaking links between particles once a certain fracture criterion is met. The main advantage of this approach is its simplicity and robustness while maintaining high accuracy at relatively low computational cost. Features such as discretization automatization and adaptivity can easily be added in meshfree methods [9,19,33].

Though—except of a simple fracture criterion—the ingredients of the method are not new, the combination of these ingredients are. The combination of these ingredients are choosen such that a robust and efficient method has been established that can handle complicated fracture patterns of arbitrarily shaped thin shells:

- To our best knowledge, it is the first time that the 3-D continuum approach based on Lagrangian kernels is applied to fracture of thin shells. Ref. [39] have shown that the choice of the kernel is essential for fracture problems and that the commonly used Eulerian kernel leads to artificial fracture.
- The simple fracture criterion within 3-D approach applied to thin structures ensures a wide application range involving complicated fracture pattern.
- The meshfree formulation ensures optimal pre- and postprocessing since no mesh is required. Thus, complicated geometries can easily be created.
- Since meshfree methods—though based on Lagrangian description of motion—can handle large deformation, fluid structure interaction can easily be incorporated. We made already a first step in this direction by studying the fluid *and* the solid, separately. The coupled system will be studied in the future.
- The implementation of the method is done in C++. We did not choose a commercial software since changes of the method can be better implemented in our own software. Moreover, due to the long history of commercial software, the data structure is often not object-oriented and flexible enough.

The efficiency of the method is demonstrated through three examples.

The paper is structured as follows: we first describe the numerical method and the constitutive model and fracture criterion. Then, we study three examples: quasi-static crack growth in order to validate our method, plate subjected to fast impact and detonation driven fracture of tube with different flaw sizes. These results are compared to experimental data. At the end, we conclude our paper and give future research directions.

2. Formulation and meshfree method

The weak form of the linear momentum in the total Lagrangian description is given as

$$\delta W = \delta W_{int} - \delta W_{ext} + \delta W_{kin} = 0 \tag{1}$$

with

$$\delta W_{int} = \int_{\Omega_0} \nabla_X \delta \mathbf{u} : \mathbf{P}\, d\Omega_0$$

$$\delta W_{ext} = \int_{\Gamma_{0t}} \delta \mathbf{u} \cdot \bar{\mathbf{t}}_0\, d\Gamma_0 + \int_{\Omega_0} \varrho_0 \delta \mathbf{u} \cdot \mathbf{b}\, d\Omega_0$$

$$\delta W_{kin} = \int_{\Omega_0} \varrho_0 \delta \mathbf{u} \cdot \ddot{\mathbf{u}}\, d\Omega_0 \tag{2}$$

where $\mathbf{b}$ denotes the body force, ϱ_0 is initial density, $\mathbf{u}$ is displacement, $\mathbf{P}$ is the first Piola–Kirchhoff stress tensor, $\bar{\mathbf{t}}_0$ is the applied traction, ∇_X denotes spatial derivatives with respect to material coordinate and superimposed dots denote material time derivatives. Above field equations are supplemented by boundary conditions:

$$\mathbf{u} = \bar{\mathbf{u}}, \quad \mathbf{X} \in \Gamma_{0u} \tag{3}$$

$$\mathbf{n}_0 \cdot \mathbf{P} = \mathbf{t}_0 = \bar{\mathbf{t}}_0, \quad \mathbf{X} \in \Gamma_{0t} \tag{4}$$

with boundaries $\Gamma_{0u} \bigcup \Gamma_{0t} = \Gamma_0$ and $\Gamma_{0u} \bigcap \Gamma_{0t} = 0$. Hereby, the index t refers to traction boundaries and the index u to displacement boundaries; $\mathbf{n}$ is the normal to the traction boundary.

The meshfree approximation $\mathbf{u}^h(\mathbf{X})$ of a given function $\mathbf{u}(\mathbf{X})$ can be expressed as the product of the shape functions with nodal parameters $\mathbf{u}_J$ as in the finite element method:

$$\mathbf{u}^h(\mathbf{X}) = \sum_{J=1}^{n} N_J(\mathbf{X})\mathbf{u}_J = \mathbf{N}\mathbf{u} \tag{5}$$

with n nodes. In the element-free Galerkin (EFG) meshfree method, the shape functions can be derived from moving least-square approximations [3] that result in the following shape functions:

$$\mathbf{N}^T(\mathbf{X}) = \mathbf{p}^T(\mathbf{X})\mathbf{A}^{-1}(\mathbf{X})\mathbf{P}\mathbf{W}(\mathbf{X}) \tag{6}$$

where is the moment matrix with moment matrix

$$\mathbf{A}(\mathbf{X}) = \mathbf{P}(\mathbf{Y})W(\mathbf{X})\mathbf{P}^T(\mathbf{Y}) \tag{7}$$

is called the moment matrix and the matrix $\mathbf{P}^T(\mathbf{Y})$ contains the polynomial basis $\mathbf{p}$ that contains polynomials up to the order of two (i.e. quadratic polynomials). Note that linear polynomial completeness is required for convergence in Galerkin methods. The matrix

$$\mathbf{W}(\mathbf{X}) = \mathrm{diag}\{W_I(\mathbf{X}-\mathbf{X}_I, h)V_I\}, \quad I = 1, \ldots, n \tag{8}$$

contains so-called kernel or weighting functions $W_I(\mathbf{X}-\mathbf{X}_I, h)$. The kernel function have compact support and the support size is determined by the dilation parameter h (e.g. the radius in circular supports). We used the quartic spline function that is commonly used in meshfree methods:

$$W(\mathbf{X}-\mathbf{X}_I, h) = w(s) = \begin{cases} 1 - 6s^2 + 8s^3 - 3s^4 & s \le 1 \\ 0 & s > 1 \end{cases} \tag{9}$$

with $s = (\mathbf{X}-\mathbf{X}_I)/2h$ for circular support size. Note that it is essential to express the kernel function in terms of material coordinates if material fracture is to be modeled appropriately [5]. More details of the formulation can be found in the meshfree literature, e.g. [3,21].