Thin-Walled Structures 49 (2011) 422–430

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Analytical calculation of local buckling and post-buckling behavior of isotropic and orthotropic stiffened panels

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ARTICLE INFO

Article history:
Received 4 May 2010
Received in revised form
27 October 2010
Accepted 11 November 2010
Available online 3 December 2010

Keywords:
Analytical solution
Energy solution
Stiffened panel
Elastic restraint edge
Local buckling
Post-buckling

ABSTRACT

A methodology for the analytical assessment of local buckling and post-buckling behavior of isotropic and orthotropic stiffened plates is presented. The approach considers the stiffened panel segment located between two stiffeners, while the remaining panel is replaced by equivalent transverse and rotational springs of varying stiffness, which act as elastic edge supports. A two-dimensional Ritz displacement function *(pb-2 Ritz)* is utilized in the solution of the local buckling problem of isotropic and laminated symmetric composite panels with arbitrary edge boundary conditions. The buckling analysis of the segment provides an accurate and conservative prediction of the panel local buckling behavior. Consequently, the developed methodology is extended in the prediction of the post-buckling response of stiffened panels of which the skin has undergone local buckling. Of high importance for the calculation of the post-buckling behavior is the selection of appropriate boundary conditions for the structural members analyzed. A comparison of the present methodology results to respective finite element (FE) results has shown a satisfactory agreement.

1. Introduction

Structures made of laminated composite materials are frequently used in engineering applications, when lightweight aspects are crucial, e.g. in aeronautical and marine structures. Composites can provide high values of specific strength and stiffness in comparison to conventional metallic materials. In many cases, thin-walled composite plates are braced by stiffening elements, in order to increase their structural efficiency and particularly their buckling behavior. For the extensive application of composite stiffened plates, the ability to solve stability problems is very important. A stiffened plate, depending on its geometry and stiffness, can exhibit buckling modes which may be divided in global and local. As sudden global buckling is undesirable, structures are usually designed such that the skin or the stiffener buckles prior to the global panel collapse; consequently, the structure enters the post-buckling regime and can still carry enough load through stress redistribution towards the un-buckled members. Especially in aerospace applications, the post-buckling load carrying capacities are often exploited in the optimization stages of lightweight structures.

The calculation of stiffened panels' global and local critical buckling load is an essential issue that has attracted the attention of several researchers. Stability problems are approached with various modeling techniques e.g. numerical [1–3], semi-analytical [4,5], energy [6], as well as analytical solutions (closed form) [8–12]. Most of the analytical–semi-analytical buckling approaches are based on the Rayleigh–Ritz method; crucial step of this method is to select an appropriate displacement function in order to properly describe the deflection of the plate in its buckled state and at the same time to satisfy the panel's boundary conditions. The existing trigonometric displacement functions can satisfy only limited types and combinations of ideal boundary conditions (e.g. free, simply supported and or clamped), hence only limited cases of isotropic and orthotropic plates have been analyzed.

Several researchers developed displacement functions to represent combinations of ideal edge condition types for solving vibration problems analytically or semi-analytically. Narita [13] has applied a trial power series function to solve the problem of free vibration response of composite rectangular plates. Liew and co-workers [14–17] have developed a two-dimensional Ritz function (pb-2 Ritz function) for general plate analysis of thin and Mindlin rectangular isotropic plates with and without internal supports.

However, a more generic displacement function is required for vibration or local buckling analysis of stiffened panels, when the elasticity between skin and stiffener is considered. Two review papers by Leissa [18,19], summarize to some extend the major studies in this field. Different vibration problems of unstiffened thin plates with rotationally restrained edges have been studied by Laura et al. [20,21], Mukhopadhyay [22], Laura and Grossi [23], Laura and Gutierrez [24], Gorman [25,26], Kobayashi and Sonoda [27], Khong and Ong [28], and Huyton and York [29].

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doi:10.1016/j.tws.2010.11.008

The methods referring to buckling analyses with elastic restraints are few [30,31] and are applied mainly to unstiffened plates. More specific, Ni et al. [30] combined two-dimensional Ritz displacement functions for an arbitrary edge condition, with HSDT plate theory; three elastic springs of constant stiffness were assumed for each edge to simulate the various edge supports. This approach was applied to symmetric laminated composite plates subjected to biaxial compression. Setoodeh and Karami [31] developed a methodology of layer-wise laminated plate theory linked to three-dimensional elasticity approach. This approach is implemented for vibration and buckling of symmetric and anti-symmetric fiber reinforced composite plates having elastically restraint edge supports.

Analytical or semi-analytical studies for calculating skin's critical local buckling load of stiffened plates using elastic restraints are very limited. Bisagni [11] developed an analytical formulation for local buckling and post-buckling analysis of isotropic and laminated stiffened plates. The structure is studied considering the part of the stiffened plate located between two stiffeners. The restraint to plate edge rotation, provided by stiffeners, is taken into account through a Saint-Venant torsion bar scheme.

Post-buckling analysis of stiffened isotropic and composite plates is considered in the literature by only few analytical and semi-analytical formulations. Stiffened isotropic plates are considered by Byklum and Amdahl [32], who developed a computational model for local post-buckling analysis; the work is based on large deflection plate theory and energy principles. Buermann et al. [33] extended the approach of Ref. [32] to the case of cylindrical isotropic stiffened plates. Both in Refs. [32] and [33] Airy stress functions are used to express the out-of-plane displacements and satisfy the compatibility equations. However, this approach is limited to isotropic materials. Romeo and Frulla [34] have studied unstiffened anisotropic plates under various boundary conditions; in a later study they have considered stiffened anisotropic plates subjected to biaxial loading with displacement control [35].

The present paper targets towards the development of a fast and efficient local buckling and post-buckling solution for stiffened panels. It is the scope of the present work to provide simple analytical solutions for the initial stages of structural design, where many optimization iterations are required for designing a structure with optimum buckling resistance. Especially in case of large-scale structures e.g. aircraft wing or ship body, which are comprised by numerous of stiffened panels, the design for buckling resistance requires semi-analytical solutions, since numerical solutions are very time-consuming and hence inefficient in the initial stages of the design.

The present buckling and post-buckling analysis are based on the classical lamination plate theory and two-dimensional Ritz displacement functions for arbitrary edge supports. An analytical tool for local buckling of isotropic and laminated symmetric composite stiffened plates is developed. The local buckling behavior of stiffened panels with different aspect ratios under axial compression is investigated. To represent the elastic edge supports, transverse and rotational springs of varying stiffness are considered along the skin-stiffener junction, allowing an accurate and conservative prediction of the critical buckling load. The longitudinal variation of stiffener's bending and torsional rigidity is selected in order to take into account the effect of the boundary conditions of the loaded edges to the skin-stiffener junction. Finally, the developed methodology is used as a tool for predicting the post-buckling response of stiffened plates after local buckling of the skin has occurred. The proposed methodology is applied in characteristic local buckling problems of isotropic and laminated symmetric composite stiffened plates. A satisfactory agreement to respective numerical results is demonstrated for all cases analyzed.

2. Local buckling analysis model

2.1. Buckling modes of isotropic and orthotropic stiffened plates

It is well known that a stiffened plate, depending on its geometry, stiffness and boundary conditions, can exhibit different buckling modes, which may be classified into global, local or mixed, as depicted in Fig. 1. In Fig. 1(a), global buckling of the plate- stiffeners assembly is shown; this mode usually occurs in stiffened plates, when the ratio of the stiffener to the skin stiffness EI_z/bD is relative small; EI_z is the stiffness of the stiffener, b is the plate width and D is the bending stiffness of the skin [36]. In Fig. 1(b), local buckling of the plate segment between the stiffeners is presented; this mode, for isotropic panels, leads to plate collapse due to local buckling and consequent yielding of the plate segment between stiffeners. In Fig. 1(c), the beam-column-type buckling of the combination of stiffener and effective plate width is represented; the failure occurs by beam-column-type collapse of the combination of stiffener and effective (reduced) plate. In Fig. 1(d), local buckling of the stiffener web is presented; this mode is usually known as "stiffener-induced" failure mode. In Fig. 1(e), lateral-torsional buckling of the stiffener web is shown, which is similar to the mode of Fig. 1(d), except that buckling of the stiffener is a lateral-torsional (tripping) buckling; once tripping occurs the stiffened plate has no effective stiffening and global buckling mode can follow immediately.

In the current work the buckling types considered are those of Fig. 1(b). Furthermore, the post-buckling response of stiffened plates exhibiting these local buckling modes is investigated.

2.2. Mathematical model and elastic edge boundary conditions

Consider a segment of the simply supported stiffened panel of Fig. 2(a), which is located between two stiffeners. Although a high number of local buckling approaches in the literature consider this segment to be either pinned or rigidly connected to the remaining stiffened panel, it is generally accepted that these behaviors are idealized edge supports, which can hardly be achieved in real practice. In practical applications, there is always some elasticity in the skin-stiffener connection area. Thus, the most appropriate boundary condition for the unloaded edges of a stiffened plate, when skin local buckling is considered, is the elastic restraint (ER) support, as depicted in Fig. 2(b).

According to the above mathematical model, the skin local buckling problem of the stiffened panel will be treated by analyzing a plate segment of uniform thickness t, length a and width b, which can be treated as simply supported (SS) in the loaded edges and elastically restrained (ER) along the longitudinal direction (x-direction), as presented in Fig. 2b. The elastic restraint supports are springs acting on the entire edge. In the present analysis the springs have extensional and rotational stiffness, which is not constant but varies along the two unloaded edges, such that they better represent the reactions of the remaining plate to the segment under analysis. As can be observed in Fig.3, the extensional springs ($\boldsymbol{k}_{11}$, $\boldsymbol{k}_{21}$) have polynomial stiffness variation along the edges (following the function of Eq. (1) below), while the rotational springs ($\boldsymbol{k}_{12}, \boldsymbol{k}_{22}$) have linearly varying stiffness along the edges (following the function of Eq. (3) below).

The stiffeners are modeled as beam elements. The assumed deflection functions are based on the deformed shape of a simply supported stiffener under bending or twisting. The extensional stiffness expressions can then be derived from the lateral deflection of the stiffener under bending as

$$k_{i1}(x) = \begin{cases} \frac{48EI_z}{x^3}, & x \le a/2 \\ \frac{48EI_z}{(a-x)^3}, & x > a/2 \end{cases} \tag{1}$$