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Forced vibration analysis of antisymmetric laminated rectangular plates with distributed patch mass using third order shear deformation theory

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ABSTRACT

A forced vibration analysis of laminated composite plates carrying a distributed patch mass is analyzed and presented in this paper. It deals with the determination of the transverse, dynamic response of rectangular composite plates subjected to a uniformly distributed $P_0e^{i\omega_{ex}t}$-type excitation. The Hamilton's Principle, using third-order shear deformation theory, is applied to simply supported rectangular plates. The displacement of the plate is postulated by a double Fourier series. The effects of size and location of the area of the patch, frequency ratio and mass ratio on the response of the plate are also presented. A thorough comparison with well-known published results is presented for the case of free vibration of unloaded plates and good agreement is observed.

1. Introduction

Vibration problems of plates are very common in engineering applications. Rectangular plates have wide applications in civil and mechanical engineering. Plates form an essential part of many aerospace, marine and automobile structures. These structural components, in many instances, are subjected to vibration. Quite frequently naval and ocean structural designers confront the problem of a plate or slab which supports a motor or an engine which excites, dynamically, the structural element. Very often the excitation is treated as a concentrated force but, obviously, it acts over a finite area, and for a better analysis, the mass of the motor or engine should be considered.

Laminated composite plates are widely used in industry and new fields of technology. Due to high degrees of anisotropy and low rigidity in transverse shear, Kirchhoff's hypothesis as a classical theory is no longer adequate. The hypothesis states that transverse normal to the mid-plane of a plate remains straight and normal after deformation because of the negligible transverse shear effects. Refined theories based on removing those restrictions of transverse normal have been recently used. As a result, the free vibration frequencies calculated by using the classical thin plate theory are higher than those obtained by Mindlin plate theory [1], in which transverse shear and rotary inertia effects are included.

A number of shear deformable theories have been proposed to date. The first such theory for laminated isotropic plates is apparently due to Stavsky [2]. The theory has been generalized to laminated anisotropic plates by Yang, Norris and Stavsky [3]. It has been shown (Sun and Whitney [4] and Bert [5] and Srinivas and Rao [6] and Serinivas et al. [7]) that the Yang–Norris–Stavsky (YNS) theory is adequate for predicting the flexural vibration response of laminated anisotropic plates in the first few modes. Whitney and Pagano [8] employed the YNS theory to study the cylindrical bending of antisymmetric cross-ply and angle-ply plate strips under sinusoidal loading and the free vibration of antisymmetric angle-ply plate strips (see also Fortier and Rossettos [9] and Sinha and Rath [10]). Bert and Chen [11] presented, using the YNS theory, a closed-form solution for the free vibration of simply supported rectangular plates of antisymmetric angle-ply laminates. Noor [12] also presented exact three-dimensional elasticity solutions for the free vibration of isotropic, orthotropic and anisotropic composite laminated plates which serves as benchmark solutions for comparison by many researchers. Free vibration of antisymmetric angle-ply laminated plates including transverse shear deformation by the finite element method was presented by Reddy [13]. Reddy [14] also derived a set of variationally consistent equilibrium equations for the kinematic models originally proposed by Reddy [14]. Reddy and Khedier [15] presented analytical and finite element solutions for vibration and buckling of laminated composite plates using various plate theories to prove necessity of shear deformation theories to predict the behavior of composite laminates. Shankara and Iyengar [16] also presented, using higher-order shear deformation theory, finite element solutions for free vibration

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analysis of laminated composite plates. Khedier and Reddy [17] obtained a complete set of linear equations of the second-order theory to analyze the free vibration behavior of cross-ply and antisymmetric angle-ply laminated plates. Singh et al. [18] presented natural frequencies of composite plates with random material properties using higher-order shear deformation theory (including rotatory inertia effect), and Kant and Swaminathan [19] presented analytical solutions for free vibration of laminated composite and sandwich plates based on higher-order refined theory. Rastgaar et al. [20] also presented natural frequencies of laminated composite plates using third-order shear deformation theory (TSDT).

While there are several reports on plate vibrations with and without added point masses, very few reports on plate vibrations with distributed mass loading can be found in the literature (Wong [23] and Kompaz and Telli [24]). Finally, though there are some papers on forced vibration of rectangular plates under harmonic loadings [25–27], a brief survey of the literature shows that forced vibration of laminated plates with distributed patch mass and excitation has not been presented.

In this paper, the forced vibration of a simply supported laminated composite plate with distributed patch mass is emphasized. It deals with the determination of the transverse, dynamic response of a rectangular composite plate subjected to a uniformly distributed $P_0 e^{i\omega_{ex}t}$-type excitation and the force acts over a rectangular portion of the plate. The problem is solved using the Hamilton's Principle by means of a double Fourier series. We present a third-order shear deformation theory, which is based on the same assumptions as the classical (CLPT) and the first-order shear deformation plate theories (FSDT), except that the assumption on the straightness and normality of the transverse normal is relaxed. Theories higher than third order are not used because the accuracy gained is so little that the effort required to solve the equations is not justified. Unlike the first order shear deformation theory, the higher order theory does not require shear correction factors. Both angle-ply and cross-ply laminates have been considered in this paper. A thorough comparison with well known published results is presented for the case of free vibration of unloaded plates and good agreement is observed.

2. Basic formulation

Consider a rectangular laminated composite plate of length a, width b and thickness h (Fig. 1). The TSDT is based on the following assumed displacement field [21]:

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) + z\varphi_x(x,y,t) - \frac{4}{3h^2}z^3\left(\varphi_x + \frac{\partial w_0}{\partial x}\right)\\
v(x,y,z,t) &= v_0(x,y,t) + z\varphi_y(x,y,t) - \frac{4}{3h^2}z^3\left(\varphi_y + \frac{\partial w_0}{\partial y}\right)\\
w(x,y,z,t) &= w_0(x,y,t)
\end{aligned}
\tag{1}
$$

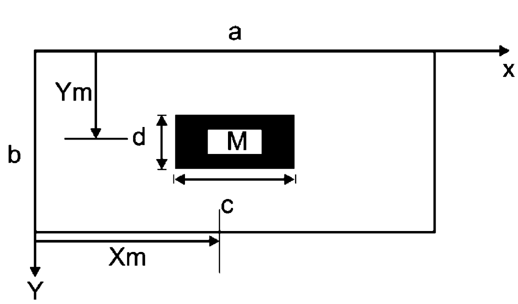

Fig. 1. Plate with distributed patch load.

u, v and w are the displacement components in the x-, y- and z-directions, respectively, u_0, v_0 and w_0 are the in plane displacements of the middle plane. φ_x and φ_y are the rotations of a transverse normal about the y and x axes, respectively. The strain-displacement equations of linear elasticity are [21]

$$
\begin{aligned}
&\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} + z\begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} + z^3\begin{Bmatrix}\varepsilon_{xx}^{(3)}\\ \varepsilon_{yy}^{(3)}\\ \gamma_{xy}^{(3)}\end{Bmatrix}\\
&\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = \begin{Bmatrix}\gamma_{yz}^{(0)}\\ \gamma_{xz}^{(0)}\end{Bmatrix} + z^2\begin{Bmatrix}\gamma_{yz}^{(2)}\\ \gamma_{xz}^{(2)}\end{Bmatrix}
\end{aligned}
\tag{2}
$$

${\varepsilon_{xx}}^{(0)}$, ${\varepsilon_{yy}}^{(0)}$, ${\varepsilon_{xx}}^{(1)}$, ${\varepsilon_{yy}}^{(1)}$, ${\varepsilon_{xx}}^{(3)}$, ${\varepsilon_{yy}}^{(3)}$, ${\gamma_{xy}}^{(0)}$, ${\gamma_{xy}}^{(1)}$, ${\gamma_{xy}}^{(3)}$, ${\gamma_{yz}}^{(0)}$, ${\gamma_{xz}}^{(0)}$, ${\gamma_{yz}}^{(2)}$ and ${\gamma_{xz}}^{(2)}$ are defined as [21]

$$
\begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial y}\\ \left(\dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}\right)\end{Bmatrix}, \quad \begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial \varphi_x}{\partial x}\\ \frac{\partial \varphi_y}{\partial y}\\ \left(\dfrac{\partial \varphi_x}{\partial y} + \dfrac{\partial \varphi_y}{\partial x}\right)\end{Bmatrix}
$$

$$
\begin{Bmatrix}\varepsilon_{xy}^{(3)}\\ \varepsilon_{xy}^{(3)}\\ \gamma_{xy}^{(3)}\end{Bmatrix} = -c_1\begin{Bmatrix}\left(\dfrac{\partial \varphi_x}{\partial x} + \dfrac{\partial^2 w_0}{\partial x^2}\right)\\ \left(\frac{\partial \varphi_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right)\\ \left(\dfrac{\partial \varphi_x}{\partial y} + \dfrac{\partial \varphi_y}{\partial x} + 2\dfrac{\partial^2 w_0}{\partial x \partial y}\right)\end{Bmatrix}
$$

$$
\begin{Bmatrix}\gamma_{yz}^{(0)}\\ \gamma_{xz}^{(0)}\end{Bmatrix} = \begin{Bmatrix}\left(\dfrac{\partial w_0}{\partial y} + \varphi_y\right)\\ \left(\dfrac{\partial w_0}{\partial x} + \varphi_x\right)\end{Bmatrix}, \quad \begin{Bmatrix}\gamma_{yz}^{(2)}\\ \gamma_{xz}^{(2)}\end{Bmatrix} = -c_2\begin{Bmatrix}\left(\dfrac{\partial w_0}{\partial y} + \varphi_y\right)\\ \left(\dfrac{\partial w_0}{\partial x} + \varphi_x\right)\end{Bmatrix}
$$

$$
c_2 = 3c_1, c_1 = 4/3h^2
$$

The constitutive relations for any layer in the (x,y) system are given by [21]

$$
\begin{aligned}
&\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix} = \begin{bmatrix}Q'_{11} & Q'_{12} & Q'_{16}\\ Q'_{12} & Q'_{22} & Q'_{26}\\ Q'_{16} & Q'_{26} & Q'_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}\\
&\begin{Bmatrix}\sigma_{yz}\\ \sigma_{xz}\end{Bmatrix} = \begin{bmatrix}Q'_{44} & Q'_{45}\\ Q'_{45} & Q'_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}
\end{aligned}
\tag{3}
$$

Q'_{ij} are the plane-stress reduced stiffness components of the layer material. The stress resultants per unit length are [21]

$$
\begin{Bmatrix}N_{\alpha\beta}\\ M_{\alpha\beta}\\ P_{\alpha\beta}\end{Bmatrix} = \int_{-h/2}^{h/2} \sigma_{\alpha\beta}\begin{Bmatrix}1\\ z\\ z^2\end{Bmatrix}dz, \begin{Bmatrix}Q_\alpha\\ R_\alpha\end{Bmatrix} = \int_{-h/2}^{h/2} \sigma_{\alpha\beta}\begin{Bmatrix}1\\ z^2\end{Bmatrix}dz \tag{4}
$$

In Eq. (4) α and β take the symbols x and y. The stress resultants are related to the strains by the relations [21]

$$
\begin{aligned}
&\begin{Bmatrix}\{N\}\\ \{M\}\\ \{P\}\end{Bmatrix} = \begin{bmatrix}[A] & [B] & [E]\\ [B] & [D] & [F]\\ [E] & [F] & [H]\end{bmatrix}\begin{Bmatrix}\{\varepsilon^{(0)}\}\\ \{\varepsilon^{(1)}\}\\ \{\varepsilon^{(3)}\}\end{Bmatrix}\\
&\begin{Bmatrix}\{Q\}\\ \{R\}\end{Bmatrix} = \begin{bmatrix}[A] & [D]\\ [D] & [F]\end{bmatrix}\begin{Bmatrix}\{\gamma^{(0)}\}\\ \{\gamma^{(2)}\}\end{Bmatrix}
\end{aligned}
\tag{5}
$$