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Mechanics of highly deformed elastic shells

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ABSTRACT

Emergence of new technological applications, in addition to the constantly growing interest in biological materials has accentuated the importance of studying the mechanics of highly deformed shells. The key challenge is the intricate interplay of physics and geometry, which leads to a mechanical response much different from the response of solid objects. The quest to understand the underlying phenomena has spawned theoretical and experimental studies, which have helped in understanding the underlying mechanisms of deformation and response of shells. Here, we use numerical simulations to study the response of shells when they are deformed deeply into the nonlinear regime. We use computational models to study the mechanics of highly deformed elastic shells in several classical problems: indentation of elastic spherical caps by a flat rigid plate and a rigid sharp indenter and pure bending of circular and oval cylinders. These assays are used to highlight some of the key aspects of the mechanics of highly deformed elastic shells, while an overview of the current state-of-the-art and suggestions for future research on this subject are also provided.

1. Introduction

The response of naturally curved elastic shells when they are highly deformed into the (geometrical) nonlinear regime is explored by focusing on two common shell configurations; cylindrical shells and spherical shells. In addition to thin shells which are normally considered for conventional materials, we have also probed the behavior and mechanics of some relatively thick shells, in view of the current interest in biological and small-scale structures. The approach here is employment of continuum-based computational models for solving shells governed by linear elasticity and fully nonlinear geometry using ABAQUS. Our material choice is restricted to that of an isotropic linear elastic material. Moreover, qualitative experiments have also been carried out on hemispherical shells to unravel some of the physical mechanisms of their response under indentation.

Shell structures have been widely used in pipelines, aerospace and marine structures, automotive industry, large dams, shell roofs, liquid-retaining structures and cooling towers [1]. Recent advancements in micro-electromechanical systems and nanotechnology have opened new avenues for applications of shells at much smaller scales. Examples are many and vary from carbon nanotubes and nanometer-sized buckyballs to microcapsules for drug delivery, colloidal armors, flexible electronics, tissue engineering and regenerative medicine [2–5]. Shell structures are also ubiquitous in nature and arise at a range of length scales from the earth's crust to microtubules and biomembranes, as well as in plants [6–8]. The current interest in understanding the behavior of living cells and subcellular components has further accentuated the importance of studying the behavior of shells. For example, much attention has been directed recently towards understanding the behavior of microtubules, which are often highly curved and buckled because of the state of stress in the cytoplasm. These studies have direct implications in understanding the physiological forces applied to microtubules, their mechanical coupling with the cytoskeleton and their role in altering cell mechanics and function [9–12]. Other examples are the mechanics of the cell membrane [13–17], the nuclear envelope [18,19] and even nanometer-sized viruses and retrovirus particles [20,21].

Despite their significance, many phenomenological aspects of the behavior of naturally curved shells are still ambiguous and pose fundamental challenges for applications of mechanics in new areas such as nanostructures and biology. Many of the nonlinear shell studies conducted in the past have been motivated by failure concerns related to conventional shell structures, and for that reason no considerable effort has been made to probe the response of these structures deep into the nonlinear regime. Emergence of novel applications at micron and submicron scales, where the material failure becomes less influential, motivates investigating the response of these structures deeply into the nonlinear regime.

The highly nonlinear behavior of elastic shells is mainly governed by inextensible or almost inextensible deformations, which are energetically preferred by the shell [22–26]. In large deformations, this leads to the appearance of structural features

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such as dimensionless developable cone and curvature condensates and almost inextensible one-dimensional ridges [27–29]. These features are ubiquitous both in nature and in technology, as well as in everyday human life. Examples are kinking of a straw and indentation of a plastic bottle using a sharp pen, as well as the crushed coke-can and dried resin. Fig. 1 provides several examples of such phenomenon in shells over a range of length scales. The interplay of physics and geometry, which indeed leads to the appearance of the localized features shown in Fig. 1, can even play a critical role at the early stage of shell response. An example is the persistence of a pinch in a circular pipe which manifests itself at the earliest stage of deformation due to the dominant role of nearly inextensible deformations [30].

In this study, first, we study the mechanics of elastic spherical caps (i.e. segment of a spherical shell) under both point-like and flat plate indentation. Then, the response of elastic circular and oval cylindrical shells under pure bending is investigated when they are deformed deep into the nonlinear regime. Our study complements a wide range of previous scaling approaches based on continuum elasticity [31–33]; and continuum-based and molecular dynamics simulations for understanding the response of highly deformed plates and shells [34,35,74]. The detailed numerical computations enable a high-fidelity exploration of highly deformed states of elastic shells.

2. Indentation of elastic spherical caps

In this section, we study the deformation and mechanics of spherical caps clamped along the edge with radius R, thickness t, and center angle α, under both flat plate and point indentations—see Fig. 2A. First, the response of these structures under rigid flat indentation is analyzed, Fig. 2. In our numerical simulations, free sliding has been assumed between the flat plate and the spherical cap. The computations were carried out with the following material parameter values: Young's modulus $E = 100$ GPa and Poisson ratio $\nu = 0.3$. Four-node shell elements with reduced integration were employed in all calculations. No initial geometric or material imperfection was included in the computational models. To follow the post-buckling response of the structure, a stabilizing mechanism based on automatic addition of volume-proportional damping was employed. For each set of calculations, the damping value was decreased systematically to assure that the response is insensitive to this change.

With Z as the deflection at the center, at the early stage of indentation, $Z<t$, the spherical cap flattens with reaction force, $F \propto Z^2$. As the indentation increases, a contact disc forms with a diameter that can be estimated from simple geometrical relation such as $2r/R = \sqrt{2Z'}$, where $Z' = Z/R$ and r denotes the disc radius. For this deformation mode, a simple scaling law based on the balance of stretching and bending energy in the spherical shell

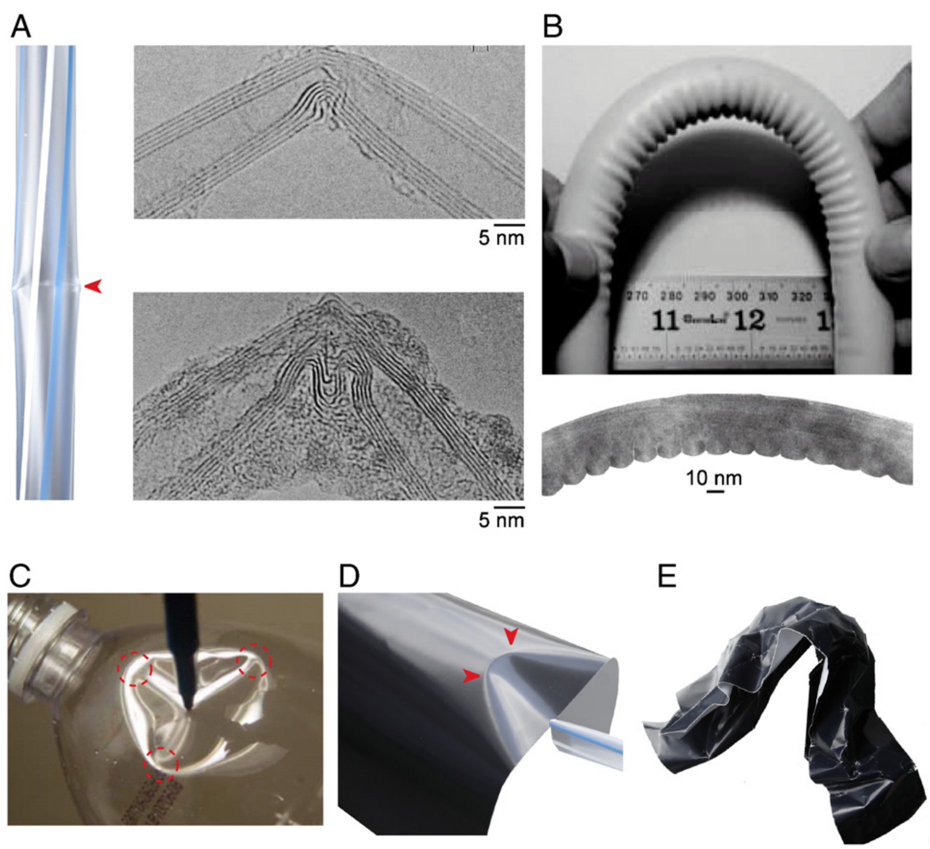

Fig. 1. (color online) *Localization of deformation in shells and plates.* (A) Kinking of cylindrical shells. Right: a straw with radius of 6 mm and an average thickness of 0.3 mm. The red arrow head denotes a kink formed upon bending the straw. Left: multi-walled carbon nanotubes with a single kink (top) and a two-kink complex (bottom) [59]. (B) Periodic rippling induced by bending a rubber tube made by rolling a thin sheet of rubber into a scroll [62], a multi-walled carbon nanotube [73]. See Mahadevan et al. [62] for further discussion. (C) Vertices formed by indentation of a plastic bottle. Under point indentation, the initial deformation mode is axisymmteric, while by increasing the indentation the deformation localizes in the form of three vertices (denoted by red circles) and approximately straight ridges—see Fig. 4. (D) Condensation of curvature in a bent paper sheet under point-like indentation. As the bent sheet is indented at its edge, a transition from the global deformation mode to a localized deformation mode occurs which leads to the condensation of curvature. Further indentation, leads to twinning, wherein the initial curvature condensate bifurcates into two (shown by the two arrow heads). See Das et al. (2007) for discussion. (E) A crumpled paper, showing a very complex deformation field comprising of vertices and folds. The paper was flat originally. (For interpretation of the references to colour in this figure legend, the reader is refered to the web version of this article.)