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Buckling analysis of thin-walled open members—A complementary energy variational principle

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Abstract

This first of two companion papers develops a new variational principle for the buckling analysis of thin-walled members based on the principle of stationary complementary energy. Some of the aspects of the Vlasov thin-walled beam theory (the rigid cross section assumption, and the stress expressions) are postulated to describe the behavior of members while other aspects of the theory (i.e., the zero shear strain assumption at mid-surface) are discarded. Koiter's formulation based on polar decomposition theory in finite elasticity is adopted to formulate expressions for statically admissible stress resultant fields. The stationarity conditions of the complementary energy expression are then evoked to yield the conditions of neutral stability and associated boundary conditions in which the rotation fields appear explicitly. The formulation seamlessly incorporates shear deformation effects and load position effects. Also, the Wagner effect and the mono-symmetry property which arise in displacement based formulations arise in the present formulation in a natural way.

Keywords: Thin-walled members; Complementary energy; Column-buckling; Lateral torsional buckling

1. Introduction and motivation

Buckling Phenomena play an important role in determining the resistance of steel members. Within this context, the present study aims at developing a new variational principle for the buckling analysis for thin-walled members of mono-symmetric cross-sections which captures shear deformation effects as well as load position effects relative to the shear center. In contrast to most buckling solutions for thin-walled members which are based on the principle of stationary potential energy, the current solution is based on the principle of stationary complementary energy which has the advantage of naturally incorporating shear deformation effects. Also, in contrast to other theories which are based on orthogonal coordinate systems, the present solution adopts a general non-orthogonal coordinate system in order to provide a natural framework to incorporate the load position effects. The solution adopts the first Vlasov assumption (i.e., the section acts as a rigid disc within the plane of the cross-section throughout deformation) and the expressions for normal and shear stresses based on the Vlasov theory, and relaxes the second Vlasov assumption (i.e., the vanishing shear strains at the section mid-surface). The variational principle is based on the polar decomposition theory in expressing the buckling stresses in the deformed configuration.

In general, global buckling is associated with long members for which shear deformation effects become negligible. Therefore, shear deformation effects have justifiably been omitted in most buckling solutions by analysts. Recent work [1] based on established shell finite element solutions has shown that for certain frame configurations (e.g., a long column supporting a short cantilever both made of wide flange sections), neglecting shear deformation effects in the cantilever portion was shown to lead to a significant overestimation of the lateral buckling load of the frame. Within this context, the present formulation attempts to capture shear deformation effects for thin-walled members while capturing lateral torsional buckling behavior.

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Nomenclature

A cross-sectional area
A_p pole (Figs. 4a and b)
a_x, a_y coordinates of arbitrary pole A_p (Fig. 4b)
$\bar{A}, \bar{S}_x, \bar{S}_y, \bar{S}_\omega$ functions to relate stress resultant functions to shear stresses
C centroid
$\mathbf{C}_{2\times 2}$ matrix due to external loading
$\mathbf{D}_{4\times 4}$ matrix of cross-sectional properties
$\mathbf{D_r}_{2\times 2}$ reduced matrix of cross-sectional properties
D_{ij} first Piola–Kirchoff stress tensor
dl_1 un-deformed length for an infinitesimal element along the principal direction
dL_1 deformed length for an infinitesimal element along the principal direction
$\vec{e}_0$ unit vectors along the principal strain directions of the *un-deformed* parallelepiped
$\vec{e}$ unit vectors along the principal strain directions of the *deformed* parallelepiped
f_j forces acting on the surfaces of infinitesimal parallelepiped
E modulus of elasticity
F_{ij} deformation gradient tensor
G shear modulus
$h(s)$ normal distance between pole and the tangent to mid-surface
I_p, I_{py}, I_{px}, $I_{p\omega}$ cross-sectional properties
$\vec{i}_{0j}$ unit vectors along X, Y and Z directions
J_d St. Venant's torsional constant
J_{xx}, J_{yy}, J_{xy}, $J_{\omega x}$, $J_{\omega y}$ section moments of inertia and products of inertia
$J_{\omega\omega}$ second moment of the sectorial coordinate
L length of a member
M_x^p, M_y^p stress resultant functions in prebuckling range
$N^p, V_x^p, V_y^p, T^p, T_w^p, T_{sv}^p$ stress resultant functions in prebuckling range
O origin
P_1, P_2, P_3, M, T forces and moments acting on a member (Fig. 1)
P_e external load potential
R_{ij} rigid body rotation tensor
r coordinate normal to the section mid surface (Fig. 5)
r_{N0} radius of gyration in non-orthogonal coordinates
s curvilinear coordinate along mid surface (Figs. 4 and 5)
S_0 sectorial origin (Fig. 4)
$S.C$ shear center
S_x, S_y, S_ω first moments of area of coordinates x, y, and ω
$t(s)$ thickness of the thin-wall segment (Fig. 5)
T total twisting moment
T_{sv} St. Venant torsion
T_w warping torsion
U^* complementary strain energy
$u(z)$, $v(z)$ displacements along x, y directions of arbitrary pole (Fig. 4a)
$u_s(s,z), v_s(s,z)$ horizontal and vertical displacements of arbitrary point B_p located on section mid-surface (Fig. 4a)
V volume of element
w Wagner stress resultant
w_i internal strain energy density
w^* complementary strain energy density
x,y,z material coordinates of a point (Fig. 2)
X, Y, Z spatial coordinates of a point (Fig. 2)
α angle between the tangent to the contour and the x axis
β_{Nx} mono-symmetry property in non-orthogonal coordinates
δ_{ij} Kronecker's delta
$\Delta N, \Delta V_x, \Delta V_y$ stress resultant increments due to buckling
$\Delta T, \Delta T_w, \Delta T_{sv} M_x, M_y$ stress resultant increments due to buckling
$\Delta u(z), \Delta v(z)$ displacements increments due to buckling along X, Y directions
$\Delta\phi_Z(z)$ rotation increment due to buckling about Z axis
ε_{ij} right extensional strain tensor
ε_{imj} permutation symbol
ϕ_X, ϕ_Y, ϕ_Z rotation vector components about X, Y and Z directions
λ buckling load factor
$\boldsymbol{\chi}_{2\times 2}$ matrix associated with the distribution of shear stress on the cross-section
ν Poisson's ratio
π^* total complementary energy
σ_{ij} Jaumann stress tensor
σ_c constant component of Jaumann stress along the thickness
σ_r vanishing component of Jaumann stress (Fig. 5)
σ_{sv} Jaumann stress due to St. Venant torsion (Fig. 5)
ω sectorial area (Fig. 4b)