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Geometric non-linear analysis of box sections under end shortening, using three different versions of the finite-strip method

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Abstract

Three finite-strip methods, (FSM) namely the semi-energy, full-energy semi-analytical and the full-energy spline FSM, are developed for predicting the geometrically non-linear response of box sections with simply supported ends when subjected to uniform end shortening in their plane. The developed FSMs are then applied to analyze the post-local-buckling behavior of some representative box sections. Although, in general, a very good agreement is observed to exist among the results obtained by all the different methods, it is revealed that in the advanced stages of post-buckling, the semi-energy method predicts results which are slightly less accurate than those obtained by the full-energy methods. This is due to the fact that a slightly higher level of compressional stiffness is experienced in the case of the results obtained by the semi-energy approach as compared to those observed in the cases of the results obtained by the full-energy methods. It is however worth noting that the current semi-energy analysis is based on a single term approach. Thus, it is expected that the accuracy of the semi-energy approach will improve and correspondingly the number of degrees of freedom involved will increase if more than one term is utilized in its formulation.

Keywords: Geometric non-linear; Post-local-buckling; Semi-energy; Finite strip; Box section

1. Introduction

Prismatic plates and plate structures are increasingly used as structural components in various branches of engineering, chief of which are aerospace and marine engineering. These structures are often employed in situations where they are subjected to in-plane compressive loading and thus it is of considerable importance to be able to accurately predict the buckling and post-buckling behavior of such structures. In aerospace, in particular, the quest for efficient, light-weight structures often leads to allowing for the possibility of local buckling and post-local-buckling behaviour. This would occur at load levels in excess of the limit conditions described by the flight envelope of an aircraft and thus the post-local-buckling reserve of load carrying capability of such structures is utilized between the limit and ultimate failure conditions of the aircraft structure.

An example of the local buckling mode of a typical strut in compression is shown in Fig. 1. As can be seen, the local buckling involves out-of-plane deflections on the web and flanges. The out-of-plane deflections grow in a stable manner as the load increases inside the post-buckling region (i.e. as the load increases beyond its critical local buckling value). The growth in the out-of-plane deflections is accompanied by continuous alterations in the stress system within the cross-section. The changes in out-of-plane deflections and the alteration in the stress system cause both the compressional and the flexural stiffness to decrease.

The post-local-buckling behavior of elastic plates or plate structures is a geometric non-linear problem. The non-linearity occurs as a result of relatively large out-of-plane deflections, which necessitates the inclusion of non-linear terms in the strain–displacement equations.

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The non-linear equations governing the elastic large deflection of flat plates were first derived by von Kármán. The post-local-buckling behavior of a plate can be analyzed by solving the von Kármán non-linear equations, together with the appropriate boundary conditions. Unfortunately, the von Kármán equations are coupled and fourth order, and thus no rigorous solutions are available. This clearly indicates that the extension of the non-linear equations from a single plate analysis to the plate structure analysis will involve even more complexity. All these have prepared the ground for the development of the approximate methods to solve the post-local-buckling problem of plates and plate structures. These approximate methods are primarily based on the principle of minimum potential energy.

Among the energy-based approximate methods, the finite-element method (FEM) has become the dominant form of geometrically non-linear structural analysis. However, although the FEM has no limitation regarding boundary conditions and local discontinuities such as openings in plates, the large number of degrees of freedom, and thus considerable computational effort required in the non-linear analysis of plates and plate structures may be considered as a deterrent factor.

For the case of prismatic structures, the finite-strip method (FSM) [1,2], which is a special form of the FEM, has proved to be a capable tool for analyzing the post-buckling behavior of plates and plate structures. As far as the computational expense is concerned, the FSM can be significantly more efficient than the FSM.

Typical examples of a plate and a strut that are modeled by finite strips are shown in Fig. 2. It is seen that the structure is divided into longitudinal strips, which are joined at longitudinal nodal lines coinciding with the strip edges.

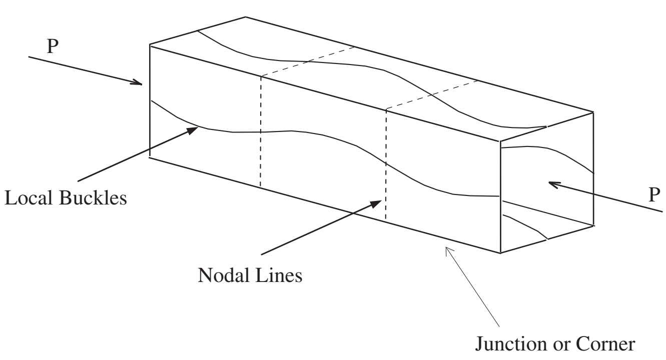

Fig. 1. A strut buckled locally.

Early works concerned with the use of the FSM in predicting the geometrically non-linear response of single rectangular plates and prismatic plate structures are those of Graves Smith and Sridharan [1–5] and Hancock [6]. These authors consider the post-buckling behavior of plates with simply supported ends when subjected to progressive end shortening. They also consider the post-buckling behavior of plate structures subjected to uniform [1–3,5] or linearly varying [4,6] end shortening, with each component plate of the structure having simply supported ends. The elastic post-buckling response of channel section struts [1] and rectangular box columns [2,3,5] have been investigated by Graves Smith and Sridharan. Hancock [6] uses the FSM to investigate the post-buckling behavior of square box and I-section columns. In the FSMs developed by the aforementioned authors, in-plane displacement fields are postulated in addition to the out-of-plane displacement field. The lengthwise variations in the displacement fields are trigonometric functions. The crosswise variations in both in-plane and out-of-plane displacement fields are simple polynomial functions. It may be noted that the above-mentioned FSMs can be categorized as semi-analytical FSM. It may also be noted that this type of FSM in which all the displacement fields are postulated by the appropriate shape functions from the onset of analysis is to be designated as the full-energy FSM in the current paper.

In the FSM developed by Graves Smith and Sridharan, the postulated forms of the longitudinal trigonometric functions are obtained with the aid of von Kármán plate equations using a perturbation technique. The displacement fields used by Graves Smith and Sridharan are appropriate for plate structures whose component plates are perfectly flat. However, this is not the case for the FSM developed by Hancock, in which slightly different displacement fields from those of Graves Smith and Sridharan are used in order to allow for the effects of the geometric imperfections to be included. The FSM developed by Hancock is appropriate only for those post-local-buckling problems in which the plate junctions are assumed not to move appreciably during the deformation process. However, the FSM developed by Graves Smith and Sridharan can handle both the aforementioned problems as well as

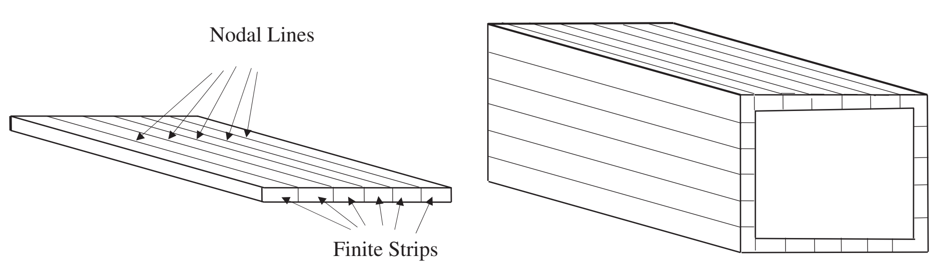

Fig. 2. Plate structures discretized by finite strips.