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Dynamic instability behavior of laminated hypar and conoid shells using a higher-order shear deformation theory

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ABSTRACT

In the present investigation, dynamic instability behavior is studied for the two laminated composite shells having twist radius of curvature, viz., hypar (hyperbolic paraboloid bounded by straight lines, HYP) and conoid (CON). A higher-order shear deformation theory is employed in the C^0 finite element formulation. Higher-order terms in the Taylor's series expansion are used to represent the higher-order transverse cross sectional deformation modes. The formulation includes Sanders' approximation for doubly curved shells considering the effect of transverse shear. The boundaries of dynamic instability regions are obtained using Bolotin's approach. The structural system is considered to be undamped. The correctness of the formulation is established by comparing the authors' results of problems with those available in the published literature. The effects of different parameters are studied on the dynamic instability regions of laminated HYP and CON shells.

1. Introduction

Shell structures are extensively used in aerospace, civil, marine and other engineering applications. In civil engineering constructions, doubly curved hypar (Fig. 1a) and conoid (Fig. 1b) shells are commonly used as roofing units. These shell forms are architecturally appealing and frequently favored for roofing large column-free areas. Hypar shells having only the twist curvature are preferred in many situations as they have ruled surface and aesthetic appeal. On the other hand, aesthetically appealing conoid shells are singly ruled surface, and hence easy to cast in addition to efficiencies in penetration of natural light. Use of composite materials in these structural components resulted in reducing the weight to increase their performance. These light weight and thin walled structural components are susceptible to a variety of time-dependent and time-independent in-plane as well as out-of-plane loads. Therefore, it is necessary to have a better understanding of their dynamic stability characteristics leading to local or global failures.

Structural elements subjected to in-plane periodic forces may induce transverse vibration. Resonance, known as parametric resonance, may occur for certain combinations of natural frequency of transverse vibration, the frequency of the in-plane forcing functions and the magnitude of the in-plane load. The spectrum of values of parameters causing unstable motion is referred to as the regions of dynamic instability.

The subject of dynamic instability had been of considerable interest since Bolotin [1] provided numerous problems on stability of structures under pulsating loads. He constructed the instability regions by using the Fourier analysis. Extensive researches on this problem and further results were reported by Evan-Iwanowski [2], Ibrahim [3] and Simitses [4]. The Lyapunov direct method was used to define the stability of a cylindrical shell under radial pressure by Bieniek et al. [5] and the solutions for the prebuckling and perturbated motions were obtained by the use of Galerkin method. Evensen and Evan-Iwanowski [6] studied the dynamic response of completely clamped shallow and thin elastic spherical shells. Yamaki and Nagai [7] investigated the dynamic stability of circular cylindrical shell subjected to periodic shearing forces on the basis of Donnell type equations modified with the transverse inertia force. Yamaki and Nagai [8] also studied the dynamic stability of circular cylindrical shells under four types of boundary conditions, using Galerkin procedure and Hsu's method. It was found that the effect of longitudinal resonance was generally negligible for thin shells. Kratzig and Eller [9] developed numerical procedures for the dynamic stability analysis of nonlinear, dynamically excited shell structures. Special algorithms were deduced for the treatment of dynamic snap-through phenomena, dynamic quasi-bifurcations and parametric resonances.

Problems of dynamic instability of composite cylindrical shells were studied by Vol'mir and Smetanina [10], Goroshko and Emel'yanenko [11] and Bondarenko and Galaka [12]. Ray [13] and

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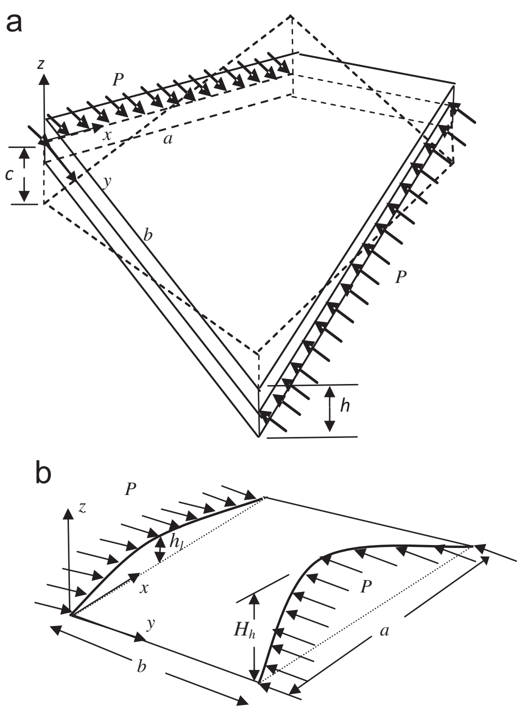

Fig. 1. Shell geometries: (a) hypar (HYP) shell and (b) conoid (CON) shell.

Ray and Bert [14] investigated parametric resonance type of dynamic stability of suddenly heated, long circular cylindrical composite shells. Effect of shear deformation of thick composite shells on their dynamic stability was investigated by Bogdanovich [15] and Bert and Birman [16]. Birman and Bert [17] studied the effect of the presence of thermal field on the dynamic stability of reinforced composite cylindrical shells subjected to pulsating loads. Cederbaum [18] analyzed parametrically excited laminated shells using the method of multiple scales (MMS). Argento and Scott [19,20] provided theoretical development and numerical results for dynamic instability of layered anisotropic cylindrical shells with clamped supports. Ganapathi et al. [21] used the finite element method to study the dynamic instability of laminated composite curved panels. Liao and Cheng [22] applied MMS to analyze the dynamic instability regions of stiffened laminated plates and shells. Lam and Ng [23] studied the dynamic stability of cylindrical shells using four different shell theories of Donnell, Love, Sanders and Flügge. Using Love's theory, Lam and Ng [24] investigated dynamic stability of laminated composite cylindrical shells subjected to conservative periodic axial loads.

Ng et al. [25–28] carried out authoritative investigations on different aspects of dynamic stability of shells. Sahu and Datta [29,30] studied parametric instability of doubly curved panels subjected to various uniform, non-uniform, partial and concentrated loadings, using Sanders' shell theory. Zhang et al. [31] studied the dynamic stability of doubly curved orthotropic shallow shells under an impact. The governing nonlinear differential equations were derived based on a Donnell type shallow shell theory. The nonlinear behavior was investigated by neglecting the influence of inertia and damping and the results showed that two-saddle node bifurcation would occur under certain conditions. Kamat et al. [32] analyzed parametrically excited laminated composite joined conical–cylindrical shells. The formulation was based on first-order shear deformation theory (FSDT) and the effects of in-plane and rotary inertia were considered. The influence of various parameters studied in the investigation included orthotropy, cone angle, layup, combination of different sections, side to thickness ratio, static load and external pressure on the dynamic instability regions of cross-ply laminates. The effect of inclusion of cutouts on the dynamic instability behavior of composite curved panels was studied by Sahu and Datta [33]. Nonlinear static and dynamic instability analyses of spherical shells were carried out by Lee et al. [34], using mixed finite element formulation. Ravi Kumar et al. [35] studied buckling, vibration and dynamic instability of laminated doubly curved panels subjected to uniaxial in-plane point and patch tensile loadings by using the finite element method. Ravi Kumar et al. [36] also studied the effect of circular cutouts on the dynamic instability characteristics of laminated doubly curved panels subjected to non-uniform tensile edge loading. Buckling and dynamic stability analyses of stiffened laminated shell panels were carried out by Patel et al. [37]. Sahu and Datta [38] provided an extensive review of most of the recent researches done in the field of dynamic instability characteristics of plates and shells in conservative systems.

It is evident from the above review that several investigators studied the dynamic instability behavior of laminated composite shells with different efficiencies and accuracies. It is observed that the dynamic instability behavior of cylindrical and spherical shells was extensively investigated by researchers. To the best of the authors' knowledge, published works on the dynamic instability of hypars and conoids are scarce. Also, the computational models employed in the earlier works mainly deal with shells with two radii of curvature $1/R_x$ and $1/R_y$ and do not account for the twist curvature $1/R_{xy}$, which is very much essential while analyzing industrially important shells like hypar and conoid. From this, it is evident that there is a need for carrying out research in the development of new computational models to study the shells having twist curvature. Therefore, in the present investigation, a higher-order theory, proposed earlier by Kant and Khare [39] for shells having two radii of curvature $1/R_x$ and $1/R_y$ is extended by including the twist radius of curvature $1/R_{xy}$ and dynamic instability behavior of industrially important hypar and conoid shells is examined by using the extended higher-order formulation.

2. Mathematical formulation

Let us consider a laminated shell made of a finite number of uniformly thick orthotropic layers (Fig. 1a), oriented arbitrarily with respect to the shell co-ordinates (x,y,z). The co-ordinate system (x,y,z) is chosen such that the plane x–y at $z=0$ coincides with the mid-plane of the shell. In order to approximate the three-dimensional elasticity problem to a two-dimensional one, the displacement components $u(x,y,z)$, $v(x,y,z)$ and $w(x,y,z)$ at any point in the shell space are expanded in Taylor's series in terms of the thickness co-ordinates. The elasticity solution indicates that the transverse shear stresses vary parabolically through the element thickness. This requires the use of a displacement field in which the in-plane displacements are expanded as cubic functions of the thickness co-ordinate. The displacement fields, which satisfy the above criteria are assumed in the form as given by Kant and Khare [39].

$$\begin{aligned} u(x,y,z) &= u_0(x,y)+z\theta_y+z^2u_0^*(x,y)+z^3\theta_y^*(x,y) \\ v(x,y,z) &= v_0(x,y)-z\theta_x+z^2v_0^*(x,y)-z^3\theta_x^*(x,y) \\ w(x,y,z) &= w_0 \end{aligned} \tag{1}$$

where u, v and w are the displacements of a general point (x,y,z) in an element of the laminate along x, y and z directions, respectively. The parameters u_0, v_0, w_0, θ_x and θ_y are the displacements and rotations of the middle plane, while u_0^*, v_0^*, θ_x^* and θ_y^* are the higher-order displacement parameters defined at the mid-plane.