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Thermo-mechanical behavior of a thin concrete shell during its early age

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Abstract

This paper reports a case study of the behavior of a concrete shell during the early stages of concrete using a two-stage computational model. First, the thermo-chemical problem is solved using a heat transfer model. The temperature evolution depends on the hydration of the concrete, which is derived from a phenomenological model. The temperatures thus obtained are next used in a mechanical model to compute the stress fields. Both models are uncoupled. The stresses are used to evaluate possible cracking of the shell. The analysis is carried out using a general-purpose finite element package for both stages of the analysis. For the case considered, the high hoop stresses computed indicate that significant meridional cracking can develop in the shell due to its early age behavior.

Keywords: Concrete shell; Early-age behavior; Finite elements; Mechanical analysis; Thermal analysis

1. Introduction

In 2001, a thin-walled reinforced concrete shell, which was the roof of a digester tank in a sewage plant, collapsed, producing considerable cost [1]. Wind and earthquake loads were ruled out as possible causes, and the investigation on the failure found that the most probable cause was the internal pressure produced by the sewage material stored inside the tank, and that the overpressures were caused by problems in the operation of the facility. Furthermore, construction of the tank had not been done according to the plans of the project, and the single layer of reinforcing bars in the dome had been displaced towards the lower part of the thickness of the shell. Failure could have occurred under those conditions, provided that there was considerable cracking in the shell prior to the overpressure. This collapse motivated interest in the evaluation of early age cracking of shells, and the results are reported in this paper.

Many problems of cracking in concrete structures are associated with thermal and shrinkage effects. The phenomena developed in the concrete, such as cement hydration, shrinkage, yield, aging and cracking, influence the behavior of the material in its early ages. Early age concrete cracking may significantly influence the durability and functionality of structures, because it increases permeability, which in turn may lead to the appearance of pore pressure, corrosion problems of steel reinforcement, filtrations, and other undesirable effects. The study of early age cracking in concrete shells is thus of essential importance to understand the real mechanisms of strength and stiffness in the structure. A detailed investigation of the stress evolution during the construction process can also be decisive to maintain the crack safety at acceptable levels.

Computational models of these early age effects have been used in recent investigations on concrete dams, and a state of the art can be found, for example, in [2–5].

This paper uses a simplification of the methodology originally developed in [5], in order to model the behavior of concrete during its early age. For this, a mathematical model has been implemented that represents the hydration and aging phenomena together with the finite element general purpose program ABAQUS [6]. This computational approach is used here to analyze the evolution of temperatures and stresses in a concrete shell in order to quantify the cracking risk to which a structure had been exposed during the process of hardening.

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The thermo-chemical model used in the simulations is discussed in Section 2, while its implementation into a general-purpose finite element program is the subject of Section 3. The application to a concrete shell is shown in Section 4. Section 5 reports results obtained from the thermal analysis, together with parametric studies. The thermo-mechanical analyses are described in Section 6. Finally, some conclusions are drawn in Section 7.

2. Thermo-chemical model

The problem of heat transfer during concrete hydration is governed by the classical heat equation, which can be written as

$$C\dot{T} - \dot{Q} = R_{ext} + k_T \nabla \cdot (\nabla T), \tag{1}$$

where C is the heat capacity per unit volume; T is the temperature; Q is an internal heat source (the rate of heat release of hydration per unit volume); R_{ext} are the external volumetric heat sources; and k_T is the thermal conductivity. Derivatives with respect to time are indicated by a dot on top of a variable. The progress of the chemical reaction during the concrete hydration is represented by means of the degree of hydration ξ, which takes values between 0 and 1, and is defined as [7]

$$\xi = \frac{\chi}{\overline{\chi}_\infty}, \tag{2}$$

where χ is the hydration amplitude and $\overline{\chi}_\infty$ the final value of the hydration amplitude in ideal conditions (that is to say, with a water–cement ratio w/c that assures the complete cement hydration). In practice, this condition is not completely fulfilled, therefore $\chi_\infty < \overline{\chi}_\infty$ and thus $\xi_\infty < 1$. The final value of the hydration degree ξ_∞ is a function of the water–cement ratio of the mixture and can be estimated according to the expression [8]:

$$\xi_\infty = \frac{1.031 w/c}{0.194 + w/c}. \tag{3}$$

Due to the thermo-activated nature of the concrete hydration reaction, the evolution of the hydration degree can be defined through an Arrhenius function in the form:

$$\dot{\xi} = \frac{k_\xi}{\eta_{\xi o}}\left(\frac{A_{\xi o}}{k_\xi \xi_\infty} + \xi\right)(\xi_\infty - \xi)\exp\left(-\overline{\eta}\frac{\xi}{\xi_\infty}\right)\exp\left(\frac{-E_a}{RT}\right) = \overline{A}_\xi(\xi)\exp\left(-\frac{E_a}{RT}\right) \geqslant 0, \tag{4}$$

where E_a is the activation energy of the reaction; R the universal constant of the gases; $\eta_{\xi o}$ the parameter that represents the initial viscosity of the material; $\overline{\eta}$ is parameter associated to the variation of the viscosity along the hydration process; k_ξ is the parameter related to the hydration rate; and $A_{\xi o}$ is the initial chemical affinity.

The function $\overline{A}_\xi(\xi)$ is a normalized affinity that characterizes the macroscopic kinetics of the hydration. A positive value indicates that the process of gel formation, as a result of the hydration reaction, is irreversible. This function can be experimentally obtained by means of an adiabatic test.

As a consequence of the chemical reaction during the hydration process, the mechanical properties of concrete evolve with time, a phenomenon called aging. It is common to characterize the change in the mechanical properties of concrete according to the compression strength.

In order to represent the aging phenomenon, the model proposed in [2,5] is used here, where the evolution of the compression strength is expressed as

$$f^-(\kappa) = \kappa f^-_\infty, \tag{5}$$

where f^- is the compression strength; f^-_∞ is its final value; and κ is the aging degree.

Next, the expressions for the evolution of the tensile strength take the form:

$$f^+(\kappa) = \lambda^+_f(\kappa) f^+_\infty = \kappa^{2/3} f^+_\infty, \tag{6}$$

while the evolution of the modulus of elasticity is obtained as

$$E(\kappa) = \lambda_E(\kappa) E_\infty = \kappa^{1/2} E_\infty, \tag{7}$$

where f^+_∞ and E_∞ are final values, i.e., when hydration has finished. Other physical properties also evolve as concrete sets, but they are taken as constants due to the lack of empirical evidence on the way they evolve.

The numerical model adopted in this paper is schematically shown in Fig. 1; this is a simplified version of the methodology proposed by [5]. In interaction 2 (thermal–hydration and aging phenomena), the expressions for thermo-chemical coupling described by [2,4] are used, but the Arrhenius law is determined for the temperature evolution in an adiabatic test, that is

$$\dot{\xi} = \overline{A_\xi}(\xi) \mathrm{e}^{-E_a/RT^{ad}}. \tag{8}$$

In interaction 4 (hydration and aging—thermal phenomena), the evolution of the internal heat source is given by

$$\dot{Q} = Q_\xi \overline{A_\xi}(\xi) \mathrm{e}^{-E_a/RT^{ad}}. \tag{9}$$

So that the evolution of the hydration degree, and consequently of the curve of internal heat generation, can be determined explicitly and independently from the heat equation and from the thermal conditions that surround the structure.

Regarding interaction 3 (hydration and aging—mechanical phenomena) the following has been used:

$$\dot{\kappa} = \lambda^-_{f,\xi}(\xi)\dot{\xi}. \tag{10}$$

Replacing Eq. (8) into Eq. (10), one gets

$$\dot{\kappa} = \lambda^-_{f,\xi}(\xi)\tilde{A}_f(\xi)\mathrm{e}^{-E_a/RT^{ad}}. \tag{11}$$

This means that the properties evolve in time with the same rate and magnitude in each point of the solid, according to the hydration degree and the temperature under an adiabatic behavior.