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Vibrations of axially moving flexible beams made of functionally graded materials

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Abstract

Problems related to the vibrations of axially moving flexible beams made of functionally graded materials are addressed. The problem of an axially moving beam may be interpreted as a telescopic system in which the mass is not constant, the mechanism of elastic deformation is transverse bending. A thin-walled beam with annular cross-section is analyzed, in which a continuously graded variation in the composition of ceramic and metal phases across the wall thickness with a simple power law is considered. In this paper a finite element scheme is employed to obtain numerical approximations to the variational equation of the problem. Normally, finite element approaches use fixed-size elements, however, for this kind of problems the increase of the number of elements, step by step as the mass enters, is a cumbersome task. For this reason an approach based on a beam-element of variable domain is adopted. The length of the element is a prescribed function of time. Results highlighting the effects of the beam flexibility, tip mass and material constituents on the dynamics of the axially moving beams are presented and the corresponding conclusions are given.

Keywords: Axially moving beams; Functionally graded materials; Non-conventional finite elements

1. Introduction

Axially moving beams appear in a broad range of problems such as telescopic robotic manipulators, deployment of flexible antennas or appendages of spacecrafts, band-saw blades, as well as the rolling process of plates, wire rods, recorder tapes and belts, among others. In this kind of problems the conservation of mass is not automatically satisfied because mass may change depending on the type of boundary conditions. That is, if the axially moving beam is considered to be inextensible or axially rigid and it is supported between two fixed points (the case of belts or band-saw blades), the mass of the system in the domain can be conserved if the motion amplitude is small. However, in the case of telescopic cantilevered beam (the case of robot arms), the mass of the system is not conserved as mass enters or leaves the domain. In this class of problems, the rate of entering mass is a prescribed value. The study of flexible beams in a translational axial movement have been gaining attention in the last years [1–7] due to new applications in the areas of robotics and spacecrafts, specifically modeling telescopic flexible actuators traveling through prismatic joints. These last applications may operate under severe environmental conditions, such as high temperatures, requiring an extended operational life. Under these circumstances, the use of functionally graded materials can offer some constructive answers in order to avoid possible structural limitations.

The functionally graded materials are a kind of composite whose properties vary continuously and smoothly from a ceramic surface to a metallic surface in a specified direction of the structure. The ceramic face protects the metallic surface from corrosion as well as thermal failure, whereas the metallic part offers strength and stiffness to the

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structure. The material properties are normally modeled varying according to a power law along the thickness of a shell [8] that constitutes the structure. The research in structural problems, focusing attention in the employment of functionally graded materials, has been mainly devoted to eigenvalue analysis of beams [9], plates and shells [8]. However, to the best of the authors's knowledge, in spite of its importance, no research work related to the vibrations of axially moving flexible beams made of functionally graded materials has been yet presented.

In the present work, a study on the vibrations of flexible sliding beams made of functionally graded materials, deployed or retrieved through a prismatic joint, is performed. The beam is modeled employing Euler–Bernoulli assumptions for small displacements and deformations [1,4]. A finite element scheme is employed to obtain numerical approximations to the variational equation of the problem. Normally, finite element approaches use fixed-size elements, however, for this kind of problems the increase of the number of elements, step by step as the mass enters, is a cumbersome task that needs a very large number of small elements in order to reach reasonable smoothness and accuracy in the results. Al-Bedoor and Khulief [6,7] developed a finite element scheme where a transition element is employed in the link as the mass enters. Although the use of transition element is an interesting idea, it presents some inconveniences in the programming stage because one has to consider that the element is partially housed in the hub, then without flexural deformation. For this reason an approach based on a beam-element of variable domain [1] is adopted in this work, where the length of the element is a prescribed function of the time. The finite element methodology is revisited in order to make clear its use in the context of a beam constructed with functionally graded materials. A study of dynamic responses for different cases of axial deploying patterns and material configurations is performed.

2. Structural model

2.1. Basic assumptions

Fig. 1 shows a horizontal flexible beam of variable length $L(t)$ moving along its longitudinal x-axis at a prescribed velocity, $V = \partial_t L(t)$. The beam has a total length L_T, and an annular cross-section, where the material properties are functionally graded in the thickness. The following hypotheses are considered in order to develop the model: (a) Bernoulli–Euler assumptions are invoked to model the structure, i.e., the cross-section is preserved from distortions in its plane, rotary inertia effects and the extensional deformation are neglected, i.e. only transverse bending is considered; (b) the gravitational potential energy due to the elastic deformations is not taken into account in comparison to the overall reference motion; (c) a tip mass is considered to be concentrated at the free end of the beam; (d) The beam is composed by ceramic and metallic phases, where a simple power-law-type definition is employed for the volume fraction of metal (ceramic) in the thickness.

The functionally graded shells are considered to be composed by many isotropic homogeneous layers [10]. The stress–strain relations for a generally isotropic material including thermal effects are expressed as [11]

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{ss} \\ \sigma_{xn} \\ \sigma_{ns} \\ \sigma_{xs} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{11} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \times \left(\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{ss} \\ \varepsilon_{xn} \\ \varepsilon_{ns} \\ \varepsilon_{xs} \end{Bmatrix} - \begin{Bmatrix} \hat{\alpha}\Delta T \\ \hat{\alpha}\Delta T \\ 0 \\ 0 \\ 0 \end{Bmatrix} \right). \tag{1}$$

The matrix elements Q_{ij} are defined in terms of effective elastic properties:

$$Q_{11} = \frac{E_{eff}}{1 - \nu_{eff}^2}, \quad Q_{12} = \frac{E_{eff}\nu_{eff}}{1 - \nu_{eff}^2}, \quad Q_{44} = Q_{55} = Q_{66} = \frac{E_{eff}}{2(1 + \nu_{eff})}, \quad \hat{\alpha} = \frac{\alpha_{eff}}{1 - \nu_{eff}}. \tag{2}$$

As the Euler–Bernoulli hypotheses are invoked, only the first two components of stress and strain of Eq. (1) would be employed.

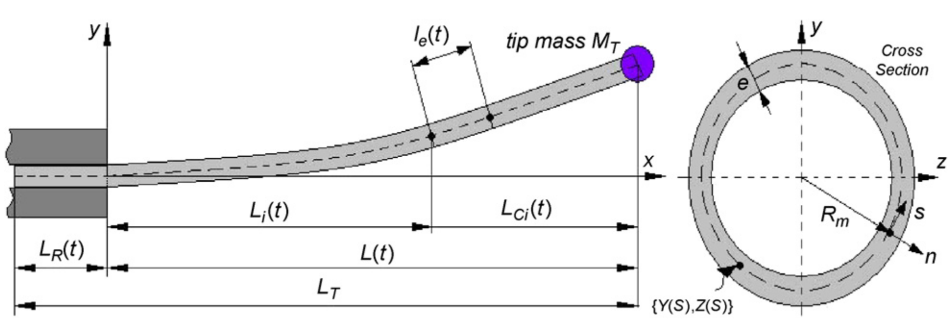

Fig. 1. Beam configuration