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Thin-Walled Structures 46 (2008) 122–127
THIN-WALLED STRUCTURES
www.elsevier.com/locate/tws

Static analysis of superelliptical clamped plates by Galerkin's method

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Received 27 November 2006; received in revised form 21 March 2007; accepted 14 August 2007
Available online 27 September 2007

Abstract

In this study, clamped superelliptical plates under uniformly distributed surface load are statically analyzed. Linearly elastic, homogeneous, and isotropic material is considered. The classical thin plate model (Kirchhoff) is employed. The lack of contributions on the static behavior of this sort of plate shapes is the fundamental motivation of the current study. Galerkin's method is used to obtain solutions. The method is conducted for polynomial series at powers ranging from 2 to 8 in order to get converging solutions. Maximum deflections of the plates and mid-point moments are obtained and the results are arranged in tabular form. For purpose of understanding, the behavior trend of the structure with respect to the parameters, some of the solutions are organized in graphical form. The study is performed for a wide range of superelliptical plates. The results are also examined with respect to the parameters a/b ratios and n, which are the plate aspect ratio and the superelliptical power, respectively.

Keywords: Plate deflection; superelliptical plates; Galerkin's method

1. Introduction

Plates which are defined by shapes between an ellipse and a rectangle have a wide range of use in engineering applications. Although we do not have sufficient engineering data, the types of the superelliptical plates that approach a rectangle with rounded corners are extensively used as structural and machine elements [1]. The studies on mechanical behavior of plates have been concentrated on rectangular, circular, and elliptical plates which can be considered as extreme cases of superelliptical plates [2–10]. Mohr [2] used a least-squares approach to obtain polynomial solutions for the deflected shape of thin plates in flexure. He worked on triangular and rectangular plates with clamped and simply supported boundary conditions and obtained solutions in good agreement with exact values which are also presented in Ref. [2]. Zenkour [3] presented a two-dimensional solution for bending analysis of simply supported functionally graded ceramic–metal rectangular sandwich plates. He obtained non-dimensional stress solutions for plates with two different ceramic–metal mixtures. The author obtained results for first-order, third-order, and classical plate theories and also worked on a sinusoidal shear deformation plate theory. The effect of material distribution on the deflections and stresses was examined. Muhammad and Singh [4] worked on a p-type solution for bending of rectangular, circular, elliptic, and skew plates. They used a first-order shear deformable plate model and obtained solutions for both clamped and simply supported plates. They also studied on plates having openings and presented their results for different solution levels and compared them with known exact values. Korol [5] worked on development of a technique for the solution of boundary value problems of the longitudinal transverse bending of orthotropic circular plates resting on a linearly elastic base. Recently, Paik [9] investigated the ultimate shear strength reduction characteristics of dented steel plates due to local impacts. In that study, a series of elastic–plastic large deflection finite element analyses were carried out for dented steel plates under edge shear loads, varying the dent size, the dent location, the plate thickness and the plate aspect ratio. Rectangular plates were considered in that study.

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doi:10.1016/j.tws.2007.08.015

2. Basic assumptions and equations

The boundary shape equation of the superelliptical plates can be represented by

$$\frac{x^{2n}}{a^{2n}}+\frac{y^{2n}}{b^{2n}}=1, \tag{1}$$

where n is the power of the super ellipse. The graphical representation of the above boundary is given in Fig. 1.

The superelliptical powers, n, are chosen from 1 to 10. For the entire study, b is kept constant as 1, and a is chosen for 14 different numbers from 1 to 20 in order to obtain results for various a/b aspect ratios.

The edges of the analyzed super ellipses are assumed to be clamped. Therefore, the solution should satisfy the following boundary conditions:

$$w=0 \quad \text{and} \quad \frac{\partial w}{\partial n_i}=0 \tag{2}$$

at the plate edge, where n_i is the outward normal of the boundary, and w is the deflection function. In addition to the geometrical boundary conditions, the kinematical boundary conditions are also satisfied by the selected trial functions. The trial functions are constructed from a complete set of polynomials in such a way that

$$w(x,y)=\sum_{i}^{r}\sum_{j}^{r}\alpha_{ij}\left(\frac{x^{2n}}{a^{2n}}+\frac{y^{2n}}{b^{2n}}-1\right)^2 x^i y^j \tag{3}$$

and $i+j\leqslant r$, so r is the order of the polynomial trial function. Here, α_{ij} are the undetermined coefficients. The existence of $((x^{2n}/a^{2n})+(y^{2n}/b^{2n})-1)^2$ in Eq. (3) guarantees that every element of these trial functions satisfies both geometrical and kinematical boundary conditions of the problem. Knowing that the deflection function of the chosen system is an even function, the elements of the trial function which has odd powers of x or y are eliminated.

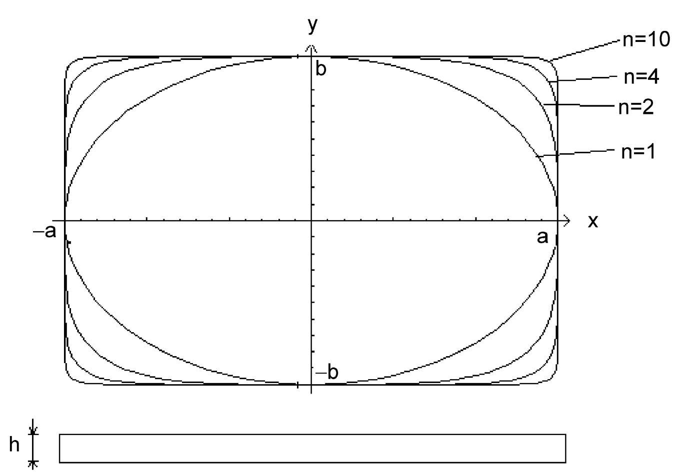

Fig. 1. Superelliptical plates having the boundary equation $(x^{2n}/a^{2n})+(y^{2n}/a^{2n})=1$.

For example, the trial function for $r=2$ is

$$w(x,y)=\left(\frac{x^{2n}}{a^{2n}}+\frac{y^{2n}}{b^{2n}}-1\right)^2\times(\alpha_{00}x^0y^0 + \alpha_{02}x^0y^2+\alpha_{20}x^2y^0). \tag{4}$$

Galerkin's method is used to obtain the solutions; therefore, the partial differential equation of the uniformly loaded plate (Eq. (5)) is directly used:

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{q_0}{D}, \tag{5}$$

where q_0 is the uniformly distributed surface load of the plate and D is the bending rigidity of the plate which is

$$D=\frac{Eh^3}{12(1-v^2)}. \tag{6}$$

Here, E is Young's modulus of the plate material, h is the plate thickness, and v is the Poisson's ratio of the plate material. Eq. (5) can also be represented in terms of biharmonic operator ∇^4:

$$\nabla^4 w-\frac{q_0}{D}=0. \tag{7}$$

3. Relation between internal forces and displacements

From Hook's law, the stress–strain relationships can be obtained as

$$\sigma_x=\frac{E}{1-v^2}(\varepsilon_x+v\varepsilon_y), \tag{8}$$

$$\sigma_y=\frac{E}{1-v^2}(\varepsilon_y+v\varepsilon_x). \tag{9}$$

The strains and stresses of the plate can now be introduced:

$$\varepsilon_x=-z\frac{\partial^2 w}{\partial x^2}, \tag{10}$$

$$\varepsilon_y=-z\frac{\partial^2 w}{\partial y^2}, \tag{11}$$

$$\sigma_x=-\frac{Ez}{1-v^2}\left(\frac{\partial^2 w}{\partial x^2}+v\frac{\partial^2 w}{\partial y^2}\right), \tag{12}$$

$$\sigma_y=-\frac{Ez}{1-v^2}\left(\frac{\partial^2 w}{\partial y^2}+v\frac{\partial^2 w}{\partial x^2}\right). \tag{13}$$

The sum of these stress components on the plate cross section produces the bending moments:

$$m_x=\int_{-(h/2)}^{+(h/2)}\sigma_x z\,\mathrm{d}z, \tag{14}$$

$$m_y=\int_{-(h/2)}^{+(h/2)}\sigma_y z\,\mathrm{d}z, \tag{15}$$