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Elastic stability of plates with circular and rectangular holes subjected to axial compression and bending moment

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ABSTRACT

In this paper linear buckling analyses of square and rectangular plates with circular and rectangular holes in various positions subjected to axial compression and bending moment are developed. The aim is to give some practical indications on the best position of the circular hole and the best position and orientation of rectangular holes in steel plates, when axial compression and bending moment act together. Two different orientations are considered for rectangular holes: holes with major dimension parallel to the vertical plate axis (RS holes) and major dimension parallel to horizontal plate axis (RL holes).

The effect of bending moment on the stability of the plate is studied and some differences with respect to the uniform compression load case are shown. Some design suggestions on the best orientation of rectangular for stability purposes are given. The influence of dimension and position of perforations on linear buckling behaviour and, in particular, on buckling coefficient of the plate is observed.

Some practical design formulations for the calculation of the buckling coefficient, taking into account (a) dimensions and shape (square and rectangular) of the plate, (b) dimensions and shape (circular and rectangular) of the hole, (c) position of the hole (centre in the “maximum” and in the “nodal” point), (d) orientation (RS and RL) of the rectangular hole, and (e) load configuration (uniform compression, combinations of axial compression and bending and pure bending) are finally proposed.

1. Introduction

The problem of stability of plates is a typical problem of steel structures, particularly bridges. A wide literature review on elastic buckling of perforated plates may be found in [1–5]. The behaviour of square shear webs having circular hole was analysed by Rockey et al. [6]; the elastic stability of bi-axially loaded rectangular plates with a single circular hole was recently studied by El-Sawy and Martini [7]. The case of concentrated loads in perforated plates was shown in [8] while axially compressed perforated plates were studied in [9,10]. Ultimate capacity of uniaxially compressed perforated plates was analysed by Narayanan and Chow [11] and strength of slender webs having eccentric holes was treated in [12]. Elasto-plastic buckling of rectangular plates in biaxial compression/tension and uniaxial compression was studied in [13,14]. Ultimate strength of perforated steel plates under shear loading was recently studied by Paik [15]. The behaviour of perforated steel plates subjected to localised symmetrical load, in the linear and non-linear phase, was recently studied by Maiorana et al. [16,17] considering the influence of the load length on stability of square and rectangular plates with circular and rectangular perforations.

A number of studies have been developed on linear buckling behaviour of unperforated steel plates, few works are available on perforated plates; in particular the common case of linear buckling behaviour of perforated plates under both axial load and bending moment is not deeply investigated. A very recent work by Komur and Sonmez [18] deals with elastic buckling of rectangular plates with a circular cutout under linearly varying in-plane normal load. In the work of Komur and Sonmez [18] only circular perforations at different locations along the horizontal x-axis are considered.

In the present work, linear buckling behaviour of square and rectangular plates with circular and rectangular perforations was studied, studying the influence of the load configuration on buckling behaviour. Circular perforations at different locations along the horizontal and vertical x- and y-axis are considered.

Emphasis is also given to the presence of rectangular holes, considering different orientations of the principal hole axis. In particular, two different orientations are considered for rectangular holes: holes with major dimension parallel to the vertical

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Nomenclature

a	width
b	height
D	flexural rigidity
E	Young's modulus
k	buckling coefficient
m	half-waves number along x
n	half-waves number along y
t	thickness
λ	slenderness
λ_1	first eigenvalue
ψ	stress ratio
σ_0	acting stress
ω	out-of-plane displacement
ν	Poisson's ratio

plate axis (RS holes) and major dimension parallel to horizontal plate axis (RL holes).

Some practical design formulations for the estimation of the buckling coefficient, depending on stress ratio ψ, are proposed for various plate aspect ratios a/b, hole dimensions d/b and hole typical locations (hole centre in "nodal" and "maximum" points) are proposed. These design formulation for the buckling coefficient are developed both for circular and rectangular perforations with RS and RL orientations.

2. Basic concepts

In this work the problem of elastic stability of square and rectangular perforated panels with simply supported edges subjected to axial compression and bending moment (Fig. 1) is studied by means of the Finite Element Code Straus7 [19]. Some design formulations for plates subjected to axial compression and bending moment are shown in the Eurocode 3 part 1–5 [20] for plates without perforations.

The collapse of plates subjected to axial load and bending moment is generally due to the loss of stability for slender panels or material yielding for thick panels. In this paper the influence of shape, dimension and position of the hole on elastic critical load of square and rectangular plates subjected to axial compression and bending moment is studied.

It is well known that linear buckling analysis is based on the determination of the buckling load through the resolution of the eigenvalue problem. The first eigenvalue λ_1 corresponds to the elastic critical load while the eigenvector defines the corresponding deformed mode.

Plate elements with four nodes (Quad4) and six degrees of freedom for each node are used in the FE study considering elements with dimensions equal to submultiples of those of the whole plate. The size of the hole and its location are the two parameters that describe the geometry of the mesh. The typical size of the element is about $b/20$ while the elements around the hole have dimensions of $b/50$ or $\pi d/40$. The four edges are simply supported; the load was applied directly to the nodes as a system of conservative forces that do not change direction after the deformation.

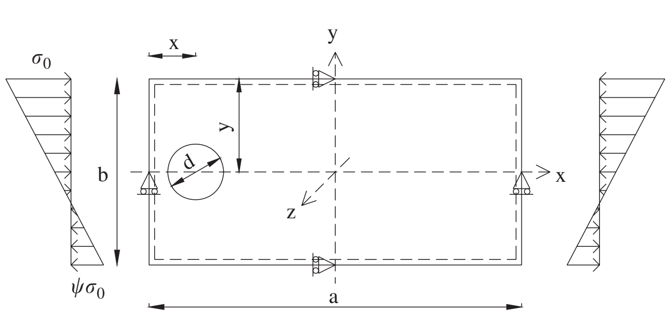

Fig. 1. Static scheme of a perforated plate subjected to axial compression and bending moment.

Buckling load of simply supported plates subjected to uniform compression is analytically determined by solving the well-known Eq. (1). The meaning of the symbols is shown in the Nomenclature. x-Axis is assumed as the horizontal axis (Fig. 1):

$$D\nabla^4\omega + \bar{N}_x \frac{\partial^2\omega}{\partial x^2} = 0 \tag{1}$$

with the following boundary conditions:

$$\omega = 0 \quad \frac{\partial^2\omega}{\partial x^2} = 0 \quad \text{along } x = 0;\ a$$

$$\omega = 0 \quad \frac{\partial^2\omega}{\partial y^2} = 0 \quad \text{along } x = 0;\ b$$

The solution has the form

$$\omega = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi x}{b} \tag{2}$$

Substituting the solution into Eq. (1) one obtains

$$A_{mn}\left[\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right) - \frac{\bar{N}_x}{D}\frac{m^2\pi^2}{a^2}\right] = 0 \tag{3}$$

Therefore

$$\bar{N}_x = \frac{\pi^2 D}{b^2}\left(m\frac{b}{a}+\frac{n^2}{m}\frac{a}{b}\right)^2 \tag{4}$$

The lower value of $\bar{N}_x$ is obtained with $n = 1$ and instability occurs with the single half-wave in y-direction for the value:

$$\bar{N}_{xC} = \frac{k\pi^2 D}{b^2} \tag{5}$$

or

$$\sigma_{xC} = \frac{\bar{N}_{xC}}{t} = k\frac{\pi^2 E}{12(1-\nu^2)}\frac{1}{(b/t)^2} \tag{6}$$

where the buckling coefficient k is

$$k = \left(m\frac{b}{a}+\frac{1}{m}\frac{a}{b}\right)^2 \tag{7}$$

Other details could be found in [21].

For the particular conditions of perforated plates considered in this work, it is possible to adopt the basic equation (6) derived for plates without perforations [17]. Significant points in the plates are considered in the following analyses. According to the critical deformed mode, the "nodal point" is assumed as the position where the sinusoidal shape does not show any out-of-plane displacement while the "maximum point" is assumed as the position where the sinusoidal shape shows the maximum out-of-plane displacement.