Thin-Walled Structures 47 (2009) 304–310

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Free vibration analysis of symmetric laminated composite plates by trigonometric shear deformation theory and inverse multiquadric RBF

Song Xiang *, Ke-Ming Wang

College of Aeroengine & Energy Engineering, Shenyang Institute of Aeronautical Engineering, No. 37, Daoyi South Avenue, Shenyang 110136, People's Republic of China

A R T I C L E I N F O

Article history:
Received 24 June 2008
Accepted 14 July 2008
Available online 30 August 2008

Keywords:
Trigonometric shear deformation theory
Laminated composite plates
Free vibration
Inverse multiquadric radial basis functions

A B S T R A C T

Based on the trigonometric shear deformation theory for laminated beams, the differential governing equations of symmetric laminated composite plates are derived. The differential governing equations discretized by a meshless collocation method based on the inverse multiquadric radial basis function is used to predict the free vibration behavior of symmetric laminated composite plates. Natural frequencies are computed for various material parameters, geometry parameters of laminated plates, and are compared with some available published results. The influence of grid pattern, modulus ratio, and side-to-thickness ratio on natural frequencies is also investigated. Through numerical experiments, the high numerical accuracy and the good convergence of the trigonometric shear deformation theory discretized by the inverse multiquadric radial basis function for free vibration analysis of symmetric laminated composite plates are demonstrated.

1. Introduction

The first-order shear deformation theory (FSDT) [1] and the higher-order shear deformation theory (HSDT) [2] for the analysis of laminated composite plates have been the subject of intense research. The FSDT and HSDT use a polynomial of z to represent the nonlinear displacement field across the thickness. This property of the displacement field leads to discontinuous transverse shear stress. Trigonometric shear deformation theories were applied to laminated composite beams by Arya [3]. These theories use a sine term to represent the nonlinear displacement field across the thickness. Transverse shear stress and strain are represented by a cosine term. This model satisfies displacement and transverse shear stress continuity at the interface. The zero transverse shear stress boundary condition at the top and bottom of the beam is also satisfied. Trigonometric shear deformation theories were also applied to static analysis for cross-ply symmetric laminated composite plates by Ferreira [4].

Free vibration analysis of laminated composite plates has been previously studied by numerous authors. Khdeir and Reddy [5] used the second-order shear deformation theory to analyze the free vibration behavior of cross-ply and anti-symmetric angle-ply laminated composite plates. Khdeir and Librescu [6] used the higher-order shear deformation theory to analyze buckling and free vibration of symmetric cross-ply elastics plates. The first-order shear deformation theory was applied to free vibration analysis of skew fibered-reinforced laminated plates by Wang [7]. Spline finite strips with the higher-order shear deformation theory were applied to static and free vibration analysis of laminated composite plates by Akhras and Li [8]. Ferreira [9] analyzed the free vibration behavior of symmetric laminated composite plates by the FSDT and radial basis functions (RBFs). Liew [10] adopted the first-order shear deformation theory in the moving least-squares differential quadrature procedure for predicting the free vibration behavior of moderately thick symmetric laminated composite plates. In their analysis, the transverse deflection and two rotations of the laminate are independently approximated with the moving least squares (MLS) approximation.

The method for solving partial differential equations includes the finite element method, the finite volume method or the finite difference method. All these methods need a mesh for local approximation. In recent years, a new method called the meshless method has been developed [11–13], in which the problem domain is discretized by a set of scattered nodes, and element connectivity among the nodes is not required. The RBFs method is a meshless method, which approximates the whole solution of the partial differential equations using RBFs. In 1990, Kansa [14,15] used the RBFs to solve partial differential equations. The multiquadrics RBFs were applied to analyze the laminated composite plates and functionally graded plates by Ferreira [16–19].

This paper focuses for the first time on the free vibration analysis of laminated composite plates by trigonometric shear deformation theories of Arya [3] and inverse multiquadric RBFs [20].

* Corresponding author. Tel.: +86 24 89723731.
E-mail address: xs74342@sina.com (S. Xiang).

0263-8231/$ - see front matter
doi:10.1016/j.tws.2008.07.007

2. Differential governing equations based on trigonometric shear deformation theories

Based on the trigonometric shear deformation theory of Arya, the displacement field of a symmetric laminated composite plate with global thickness h can be defined as

$$\begin{aligned} u &= -z\frac{\partial w(x,y)}{\partial x} + \sin\frac{\pi z}{h}\phi_x(x,y) \\ v &= -z\frac{\partial w(x,y)}{\partial y} + \sin\frac{\pi z}{h}\phi_y(x,y) \\ w &= w(x,y) \end{aligned} \tag{1}$$

where w is the deflection, and ϕ_x and ϕ_y are the rotations of the normal to the mid-plane about the y and x axes, respectively.

The strain–displacement relationships are given by

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{Bmatrix} \tag{2}$$

By substituting Eq. (1) into Eq. (2), the strains can be expressed as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = z\begin{Bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ -2\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} + \sin\frac{\pi z}{h}\begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \frac{\pi}{h}\cos\frac{\pi z}{h}\begin{Bmatrix} \phi_y \\ \phi_x \end{Bmatrix} \tag{4}$$

The stress–strain relationships in the global x–y–z coordinate system can be written as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{5}$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \\ \bar{Q}_{12} &= (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta+\cos^4\theta) \\ \bar{Q}_{16} &= (Q_{11}-Q_{12}-2Q_{66})\sin\theta\cos^3\theta + (Q_{12}-Q_{22}+2Q_{66})\sin\theta^3\cos\theta \\ \bar{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \\ \bar{Q}_{26} &= (Q_{11}-Q_{12}-2Q_{66})\sin\theta^3\cos\theta + (Q_{12}-Q_{22}+2Q_{66})\sin\theta\cos^3\theta \\ \bar{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta+\cos^4\theta) \\ \bar{Q}_{44} &= Q_{44}\cos^2\theta + Q_{55}\sin^2\theta \\ \bar{Q}_{45} &= (Q_{55}-Q_{44})\cos\theta\sin\theta \\ \bar{Q}_{55} &= Q_{55}\cos^2\theta + Q_{44}\sin^2\theta \end{aligned} \tag{6}$$

where θ is the angle between the 1-axis and the x-axis, 1-axis being the first principal material axis, and the reduced stiffness components are given as

$$\begin{aligned} &Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \nu_{21}Q_{11} \\ &Q_{66} = G_{12}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13}, \quad \nu_{21}E_1 = \nu_{12}E_2 \end{aligned} \tag{7}$$

where E_{ij}, ν_{ij}, and G_{ij} are Young's modulus, Poisson ration, and shear modulus, respectively.

Euler–Lagrange equations, derived by using the principle of virtual work, are

$$\begin{aligned} &\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} = I_1\frac{\partial^2 w}{\partial t^2} \\ &\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{\pi}{h}Tc_{xz} = I_2\frac{\partial^2 \phi_x}{\partial t^2} \\ &\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} - \frac{\pi}{h}Tc_{yz} = I_2\frac{\partial^2 \phi_y}{\partial t^2} \end{aligned} \tag{8}$$

where

$$\begin{Bmatrix} N_{\alpha\beta} \\ M_{\alpha\beta} \end{Bmatrix} = \int_{-h/2}^{h/2} \sigma_{\alpha\beta}\begin{Bmatrix} \sin\frac{\pi z}{h} \\ z \end{Bmatrix} dz \tag{9}$$

$$Tc_{\alpha z} = \int_{-h/2}^{h/2} \sigma_{\alpha z}\cos\frac{\pi z}{h}\, dz \tag{10}$$

where α and β denote the symbols x and y, and I_i are the mass inertias defined as

$$I_1 = \int_{-(h/2)}^{h/2} \rho\, dz, \quad I_2 = \int_{-(h/2)}^{h/2} \rho z^2\, dz$$

where ρ denotes the material density.

The differential governing equations can be obtained by substituting Eqs. (9) and (10) into Eq. (8) as

$$\begin{aligned} &zs_{11}\frac{\partial^3 \phi_x}{\partial x^3} + (zs_{12}+2zs_{66})\frac{\partial^3 \phi_x}{\partial x \partial y^2} + (zs_{16}+2zz_{16})\frac{\partial^3 \phi_x}{\partial x^2 \partial y} \\ &+ zs_{26}\frac{\partial^3 \phi_x}{\partial y^3} + (zs_{12}+2zs_{66})\frac{\partial^3 \phi_y}{\partial x^2 \partial y} + zs_{22}\frac{\partial^3 \phi_y}{\partial y^3} + zs_{16}\frac{\partial^3 \phi_y}{\partial x^3} \\ &+ 3zs_{26}\frac{\partial^3 \phi_y}{\partial x \partial y^2} - zz_{11}\frac{\partial^4 w}{\partial x^4} - zz_{22}\frac{\partial^4 w}{\partial y^4} - 2zz_{12}\frac{\partial^4 w}{\partial x^2 \partial y^2} \\ &- 4zz_{66}\frac{\partial^4 w}{\partial x^2 \partial y^2} - 4zz_{16}\frac{\partial^4 w}{\partial x^3 \partial y} - 4zz_{26}\frac{\partial^4 w}{\partial x \partial y^3} = I_1\frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{11}$$

$$\begin{aligned} &ss_{11}\frac{\partial^2 \phi_x}{\partial x^2} + ss_{66}\frac{\partial^2 \phi_x}{\partial y^2} - cc_{55}\left(\frac{\pi}{h}\right)^2\phi_x + 2ss_{16}\frac{\partial^2 \phi_x}{\partial x \partial y} + ss_{16}\frac{\partial^2 \phi_y}{\partial x^2} \\ &+ (ss_{12}+ss_{66})\frac{\partial^2 \phi_y}{\partial x \partial y} + ss_{26}\frac{\partial^2 \phi_y}{\partial y^2} - cc_{45}\left(\frac{\pi}{h}\right)^2\phi_y - zs_{11}\frac{\partial^3 w}{\partial x^3} \\ &- (zs_{12}+2zs_{66})\frac{\partial^3 w}{\partial x \partial y^2} - 2zs_{16}\frac{\partial^2 w}{\partial x \partial y} - zs_{16}\frac{\partial^3 w}{\partial x^2 \partial y} \\ &- zs_{26}\frac{\partial^3 w}{\partial y^3} = I_2\frac{\partial^2 \phi_x}{\partial t^2} \end{aligned} \tag{12}$$

$$\begin{aligned} &ss_{12}\frac{\partial^2 \phi_x}{\partial x \partial y} + ss_{66}\frac{\partial^2 \phi_x}{\partial x \partial y} + ss_{26}\frac{\partial^2 \phi_x}{\partial y^2} + ss_{16}\frac{\partial^2 \phi_x}{\partial x^2} - cc_{45}\left(\frac{\pi}{h}\right)^2\phi_x \\ &+ ss_{22}\frac{\partial^2 \phi_y}{\partial y^2} + ss_{66}\frac{\partial^2 \phi_y}{\partial x^2} - cc_{44}\left(\frac{\pi}{h}\right)^2\phi_y + 2ss_{26}\frac{\partial^2 \phi_y}{\partial x \partial y} \\ &- zs_{22}\frac{\partial^3 w}{\partial y^3} - (zs_{12}+2zs_{66})\frac{\partial^3 w}{\partial x^2 \partial y} - 3zs_{26}\frac{\partial^3 w}{\partial x \partial y^2} \\ &- zs_{16}\frac{\partial^3 w}{\partial x^3} = I_2\frac{\partial^2 \phi_y}{\partial t^2} \end{aligned} \tag{13}$$