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The effect of geometric imperfections on the vibrations of anisotropic cylindrical shells

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Abstract

Analytical–numerical models to analyse the flexural vibration behaviour of anisotropic cylindrical shells are presented. The two models (denoted as Level-1 and Level-2 Analysis) have different levels of complexity and can be used to study the influence of important parameters, such as geometric imperfections, static loading, and boundary conditions. A specific anisotropic shell is used in the calculations in this paper. The influence of the imperfection shape and amplitude on the natural frequency is investigated for this shell via both the Level-1 and the Level-2 Analysis. Imperfections with the shape of the "lowest vibration mode" give a decrease of the natural frequency with increasing imperfection amplitude. The results of the Level-2 Analysis for the effect of imperfections on the natural frequency are in reasonable agreement with Finite Element calculations.

Keywords: Vibration; Shell structures; Geometric imperfections

1. Introduction

The vibration behaviour of imperfection-sensitive structures has received attention in several works. Firstly, as is nowadays widely known, unavoidable small deviations from the perfect form due to the fabrication process (the so-called geometric imperfections) can considerably affect the buckling behaviour of shell structures (see e.g. the review article [1]). This factor, which can have such an important effect on the buckling behaviour, can also influence the (linear and nonlinear) dynamic behaviour of shell structures. Secondly, the formal analogy between buckling and vibration has stimulated the use of vibration tests to obtain information which is important to assess the buckling behaviour. A first method is that one could use vibration tests to establish the actual boundary conditions, the so-called vibration correlation technique [2]. Another possibility is to estimate the buckling load from vibration tests as a way of nondestructive testing, see e.g. [3]. A good understanding of the effects caused by imperfections on the vibration behaviour is indispensable when such methods are applied.

The vibration behaviour of imperfect spherical shells was investigated in [4], while in [5] imperfect oval cylindrical shells were analysed. Investigations on linear and/or nonlinear vibrations of cylindrical panels in which the effect of imperfections is included are described in e.g. [6–10].

Due to its theoretical and practical significance, the circular cylindrical shell is of special interest. The influence of axisymmetric imperfections and asymmetric imperfections on the natural frequencies of circular cylindrical shells was studied by Rosen and Singer [11,12] for isotropic shells and later by Singer and Prucz for orthotropic shells [13]. The effect of imperfections was included by Watawala and Nash [14] in the nonlinear (i.e. large amplitude) vibration analysis of isotropic shells, and by Liu [15] in the analysis of orthotropic shells. Liu addressed the discrepancy between the results of [12,14,16] regarding the effect of asymmetric imperfections on the linearized vibration behaviour of isotropic shells. Liu confirmed the results obtained by Hol [16], who predicted a decrease in frequency with increasing imperfection amplitude for a typical shell with an asymmetric imperfection affine to the

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vibration mode. More recently, Amabili [17] investigated the effect of different boundary conditions on the nonlinear vibrations of cylindrical shells. The effect of imperfections was included in his model. An extensive review on nonlinear vibrations of cylindrical shells and panels can be found in [18].

In the present paper, models with different levels of complexity are used to analyse the vibration behaviour of anisotropic (such as composite) cylindrical shells [19]. Two types of analytical–numerical models will be presented. With these models the influence of important parameters, such as large amplitudes, geometric imperfections, static loading, and boundary conditions can be investigated. Results of these models for the large amplitude vibration analysis have been presented for isotropic and orthotropic shells in [20] and for anisotropic shells in [21].

The use of laminated structures requires an analysis which takes into account the possible couplings between the bending and stretching deformations. Nonlinear Donnell-type governing equations are adopted in combination with classical lamination theory. It is assumed that the cylindrical shell is statically loaded by axial compression, radial pressure, and torsion.

In a Level-1 Analysis (or Simplified Analysis) a small number of assumed modes which approximately satisfy "simply supported" boundary conditions at the shell edges, are used in a Galerkin procedure or variational method. In a Level-2 Analysis (or Extended Analysis) the specified boundary conditions are accurately satisfied by means of the numerical solution of corresponding two-point boundary value problems for ordinary differential equations.

The aim of the present paper is to show the capability of these analytical–numerical models by analysing the small amplitude vibration behaviour of imperfect anisotropic shells. In particular, the possibility to analyse anisotropic cylindrical shells is shown through the example of a specific composite shell. Attention will paid to the discrepancy in the literature between the results for the effect of asymmetric imperfections referred to earlier. A comparison with Level-3 Analysis (Finite Element) results will be made.

2. Governing equations

The shell geometry and the applied loading are defined in Fig. 1. The shell geometry is characterized by its length L, radius R and thickness h. The constitutive equations for a laminated shell can be written as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}, \quad (1)$$

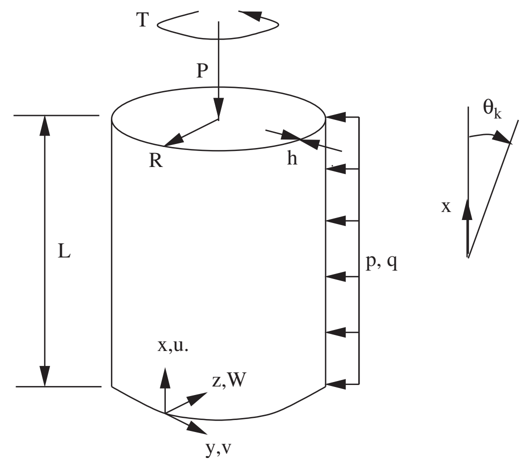

Fig. 1. Shell geometry, coordinate system and applied loading.

$$\begin{bmatrix} M_x \\ M_y \\ \frac{M_{xy}+M_{yx}}{2} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}, \quad (2)$$

where N_x, N_y and N_{xy} are the usual stress resultants, M_x, M_y, M_{xy} and M_{yx} the moment resultants, ε_x, ε_y and γ_{xy} the strains, and κ_x, κ_y and κ_{xy} the curvatures. The stiffness coefficients A_{ij}, B_{ij} and D_{ij} $(i,j = 1,2,6)$ from classical lamination theory are used. The definition of the layer orientation can be found in Fig. 1.

The constitutive equations, Eqs. (1) and (2) can be written in matrix form as

$$\{N\} = A\{\varepsilon\} + B\{\kappa\}, \quad (3)$$

$$\{M\} = B\{\varepsilon\} + D\{\kappa\}, \quad (4)$$

and after partial inversion as

$$\{\varepsilon\} = A^*\{N\} + B^*\{\kappa\}, \quad (5)$$

$$\{M\} = C^*\{N\} + D^*\{\kappa\}, \quad (6)$$

where

$$A^* = A^{-1},$$

$$B^* = -A^{-1}B,$$

$$C^* = BA^{-1} = -B^{*T},$$

$$D^* = D - BA^{-1}B.$$

Assuming that the radial displacement W is positive inward (see Fig. 1) and introducing an Airy stress function F as $N_x = F_{,yy}$, $N_y = F_{,xx}$ and $N_{xy} = -F_{,xy}$, where $()_,$ denotes partial differentiation with respect to the variable following the comma, then the Donnell-type nonlinear imperfect shell equations (neglecting in-plane inertia) for a general