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Structural optimization of thin-walled tubular trusses using a virtual strain energy density approach

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Abstract

The present paper deals with the weight minimization of tubular trusses subjected to multiple loads under size, stress and buckling constraints. The applied optimization procedure is based on a virtual strain energy density approach developed by the first two authors, already tested in plane and space truss structures. The key point of the method is the activation of at least one of the imposed displacement constraints. In case where such limitations are absent, a dummy displacement constraint is introduced instead, which iteratively sustains corrections until convergence is achieved within the desirable tolerance. The efficiency and practicability of the proposed method was tested in typical cases of tubular truss structures. For reasons of comparison, the same cases were also optimized using Sequential Quadratic Programming (SQP), which is a powerful mathematical programming optimization method. The results revealed that the proposed method performs very well in terms of convergence, of required number of iterations and of optimum tracing, while the value of the introduced dummy displacement constraint has insignificant effect on the optimization procedure.

Keywords: Optimization; Optimality; Tubular structure; Buckling; Finite element

1. Introduction

Morris [1] gives a detailed review mainly in the field of optimality criteria (OC) and of mathematical programming (MP) methods until 1982. The fully stressed design (FSD) is among the most simple and popular (OC) methods. Nevertheless, its inability to confront with displacement constraints, which are typical design limitations of modern structures, in combination with the fact that not always does it provide the optimum design for a stressed constrained structure, detracts some of its value. Gellatly and Berke [2], using optimality criteria, proposed a recursion expression for the design of structures with displacement constraints. Based on the sign of the virtual strain energy, they separated the members in 'active' and 'passive'. The active members have a positive virtual strain energy value and are allowed to be redesigned, while the passive members have a negative virtual strain energy value, they are excluded from the redesign process and their minimum size has to satisfy stress or fabrication requirements. Research was also turned to combining (OC) methods with other optimization procedures [3–6]. In more details, Fleury and Geradin [3] combined optimality criteria with pure mathematical programming, thus allowing the convergence control of the optimization process. Allwood and Chang [4] presented an optimality criteria procedure based on the Newton–Raphson method, thus improving the method of Taig and Kerr [5]. More particularly, their procedure involved the iterative solution of a non-linear system of equations, where the primary variables were Lagrange multipliers. Patnaik et al. [6] modified the Fully Utilized Design (FUD) method, which is the extension of the traditional fully stress design (FSD) method including displacement limitations in addition to stress constraints. Viewing the literature [6–17], it seems that there is still place for ideas with better application properties in this field.

Towards this direction, the present paper proposes the use of the iterative redesign formula presented in its initial form in 1995 [15] and in its updated expression in 2002 [16] for the case of tubular pipes, where the two design variables sustain geometrical limitations. The formula contains the product of two independent multipliers. The first multiplier is defined as

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the ratio of the virtual strain energy density of a member over the sum of the absolute values of the virtual strain energy densities of all members, without the distinction between active and passive members being made. The second multiplier is defined as the ratio of the largest displacement violation over the value of the corresponding displacement constraint. After the application of the redesign formula, it is checked whether the cross-section values violate the minimum allowable ones. In the sequel, the stresses are checked and a correction takes place only for the cross-section of those members experiencing larger stress than the allowable. The aforementioned redesign procedure assumes that at least one displacement constraint is active. If this does not occur, or if no displacement constraints are imposed, then the redesign procedure is still applicable by introducing a dummy constraint, as it is shown in the present work.

2. The proposed recursive formula

In numerous optimization methods, the principle of virtual work plays a significant role in finding the optimal design. In the case of a truss, the virtual work (left-hand side of Eq. (1)) is the product of a virtual load Q applied to a node of the truss, multiplied by the corresponding caused nodal displacement u which is collinear to the force:

$$Qu = \sum_{i=1}^{n} \frac{F_i^P F_i^Q L_i}{E_i A_i} \tag{1}$$

A_i and L_i are the area and the length, respectively, of the ith truss member (bar), where F_i^Q and F_i^P are the forces developed in the i-th bar due to the application of the virtual load Q and of the external loads P, respectively. E_i is the elastic modulus of the material and n is the total number of the truss members.

When a virtual unit nodal load ($Q=1$) is applied, then Eq. (1) takes the following form:

$$u = \sum_{i=1}^{n} u_i = \sum_{i=1}^{n} \frac{F_i^P F_i^Q L_i}{E_i A_i} = \sum_{i=1}^{n} \left(\sigma_i^P \sigma_i^Q \frac{V_i}{E_i} \right) \tag{2}$$

The term u_i has a two-fold interpretation, the former being the virtual strain energy characterizing the i-th bar and the latter being the contribution of the i-th bar to the displacement u of the node, which the virtual unit load ($Q=1$) is applied to. This information is valuable in case where a cross-section redesign is desirable so that the nodal displacement u approaches a limiting value u_0. The corresponding virtual strain energy per unit volume, or Virtual Energy Density (VED), of the i-th bar is defined as follows:

$$U_i \equiv \frac{u_i}{V_i} = \frac{F_i^P F_i^Q}{E A_i^2} \tag{3}$$

For a determinate truss, both forces F_i^P and F_i^Q are independent of the cross-sectional area A_i, therefore it holds that

$$u_i = \frac{c_i}{A_i} \quad \text{with} \quad c_i = \frac{F_i^P F_i^Q L_i}{E} = \text{const.} \tag{4}$$

Differentiation of Eq. (4) gives:

$$\frac{du_i}{dA_i} = -\frac{c_i}{A_i^2} \tag{5}$$

Division of Eqs. (4) and (5) by parts yields:

$$\frac{du_i}{u_i} = -\frac{dA_i}{A_i} \tag{6}$$

Eq. (6) equates the rate of change of the cross-sectional area of the i-th bar to the rate of change of the contribution that the i-th bar has on the displacement of the node which the virtual load is applied to. Using small finite variations Δu_i and ΔA_i, Eq. (6) can be written as follows

$$\frac{\Delta u_i}{u_i} = -\frac{\Delta A_i}{A_i} \tag{7}$$

or equivalently as:

$$A_i^{\text{new}} = A_i^{\text{old}} \left(1 + \frac{\Delta u_i}{u_i} \right) \tag{8}$$

According to the proposed method, the following estimation is used (for more details, see [16])

$$\frac{\Delta u_i}{u_i} = \eta_i \frac{\Delta u}{u}, \quad i = 1,2,\ldots,N \tag{9}$$

where

$$\eta_i = \frac{U_i}{\overline{U}} = \frac{U_i}{\frac{1}{N}\sum_{j=1}^{N} |U_j|} \tag{10}$$

and

$$\Delta u = u - u_0 = \sum_{i=1}^{N} \Delta u_i \tag{11}$$

The combination of Eqs. (8)–(11) results in the following redesign formula:

$$A_i^{\text{new}} = A_i^{\text{old}} \left[1 + \eta_i \frac{u - u_0}{u_0} \right] \tag{12}$$

In more details:

(a) If $\eta_i > 0$ (equivalently $u_i > 0$), then according to Eq. (2) the i-th bar contributes in a positive way to the formation of the displacement u. In this case, Eq. (12) suggests that if the current displacement violates the constraint ($(u-u_0)/u_0 > 0$), we can further decrease it by increasing the value of the cross-section A_i. On the contrary, if the current displacement is below the limit ($(u-u_0)/u_0 < 0$), we can safely increase it by decreasing the value of the cross-section A_i.

(b) If $\eta_i < 0$ (equivalently $u_i < 0$), then the i-th bar resists to the increase of the total displacement. Therefore, if the