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# Crashworthiness design of multi-corner thin-walled columns

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ABSTRACT

This paper presents a crashworthiness design of regular multi-corner thin-walled columns with different types of cross-sections and different profiles, including straight octagonal columns and curved hexagonal columns. In this paper, the straight octagonal section columns are first optimized, which mainly take axial crash loads during crashes. Next, the curved hexagonal section columns are optimized following the same approach, which are subject to bending moment when impact occurs. During the design optimizations, specific energy absorption (*SEA*) is set as the design objective, side length of the cross-sections and wall thickness are selected as design variables, and maximum crushing force ($P_m$) is set as the design constraint. Both the objective and constraint are formulated using the response surface method (RSM) based on sets of finite element (FE) results obtained from FE analyses (FEA). After obtaining the optimal designs, parametric studies are performed to investigate the influences of the design variables on the crash performance of such multi-corner thin-walled columns.


## 1. Introduction

The use of thin-walled multi-corner columns (with hexagonal and octagonal cross-sections) is growing continuously in civil engineering, automotive engineering, shipbuilding, and other industries, because of their high strength–weight ratio, low cost, and excellent energy-absorption capability during crashworthiness analysis. Such columns most appear in truss and frame structures as major energy-absorbing components and absorb a substantial amount of crash energy when the impact occurs. Therefore, such columns receive a lot of research interests and previous literatures have demonstrated their responses and performances during crashworthiness analyses [1–3,12,13]. In designing such columns, maximizing their energy-absorption capability should always be a major objective. As presented in previous researches, there are two approaches to enhance the performance of the multi-corner thin-walled columns: either using advanced materials with high mechanical properties [4,5] or designing optimized wall thickness and cross-sectional dimensions for such columns that can provide the best crash performances [6]. Hou and other co-researchers [6] presented the optimal designs of straight hexagonal thin-walled columns with singly celled and triply celled configurations, which provided the maximum energy-absorption capability during the crashworthiness analyses. However, little effort has been spent on the optimization of the cross-sectional dimensions of the octagonal thin-walled columns. Thus, in this article, an optimum design is first performed for the cross-sectional profiles of such columns to maximize their capability of energy absorption. Also, besides the straight columns, curved thin-walled column is another important energy-absorbing component which mainly takes bending moment during the crashes. Now that Hou [6] has derived the optimal design for the straight hexagonal columns, this paper continues to employ the same design method to obtain the optimal design for the curved hexagonal columns.

During the optimum design, an advanced technique, the response surface method (RSM) is applied to approximately formulate the columns' energy-absorption capabilities. The RSM is presented by Myers and Montgomery and extensively developed by other researchers [7,8], which is to use some simple basis functions such as polynomials to approximate the crash behavior of a structure. This method has been employed to optimize several other thin-walled structures with crashworthiness criterion [4,9–11]. In those studies, the polynomial basis functions were used to model the energy absorption. On the basis of the previous researches, the RSM with the polynomial basis functions are used in this paper to obtain the optimum design for the thin-walled octagonal section columns.

To seek for the optimal crashworthiness design a set of designs are sampled from the design space using the factorial design, which have different cross-sectional dimensions. Finite element (FE) models are created for those designs and used for computer crashworthiness analyses to provide crash responses of those design samples, based on which the RSMs are constructed . Next, the optimal design for the curved hexagonal columns is derived

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following the same design approach. The optimized hexagonal cross-section for the curved columns obtained in this study is compared to the one for the straight columns, which was presented by Hou [6] to compare the crashworthiness designs for straight and curved thin-walled columns.

Besides the design optimization, parametric studies are performed to investigate the influences of the cross-sectional dimensions on the columns' crash performances. In this project, the implicit solver ANSYS is used to build up the geometric beam models and the explicit solver LS-DYNA is used to generate the FE models and perform the crashworthiness analyses.

## 2. Problem description

In crashworthiness design the energy-absorption capacity of a thin-walled column is measured by the specific energy absorption (*SEA*), which is the crash energy absorbed per unit weight of the thin-walled column. Therefore, the major objective of this optimum design is to maximize the *SEA* for the octagonal thin-walled columns. The *SEA* is defined as

$$SEA = Total\ absorbed\ energy\ E_{\text{total}}/Total\ structural\ weight \tag{1}$$

Two factors have to be considered during this design. At first, based on the human safety issues, the maximum crushing force $P_{\text{m}}$ that occurs during the crash should not exceed a certain criteria, which is an important issue in the vehicle design and manufacturing. Also, the two design variables of the optimized octagonal cross-section, its side length $a$ and wall thickness $t$ (Fig. 1), only vary between their upper and lower bounds. Thus, this optimization problem is formulated as

$$\begin{aligned}&\text{Maximize}: SEA(a,t)\\ &\text{Constraints}: P_{\text{m}} \leqslant \text{Criteria},\ a^{\text{L}} \leqslant a \leqslant a^{\text{U}} \text{ and } t^{\text{L}} \leqslant t \leqslant t^{\text{U}}\end{aligned} \tag{2}$$

where $a^{\text{L}}$, $a^{\text{U}}$, $t^{\text{L}}$, and $t^{\text{U}}$ are the lower and upper bounds of the design variables $a$ and $t$, respectively.

## 3. Response surface method [7]

RSM is extensively applied in the modern industry for developing, improving, and optimizing processes. During a design process such a method is used to determine several input variables (independent variables), which potentially influenced the performance or quality of the system in order to achieve optimized responses from that system. In this study, RSM is employed to determine the $a$ and $t$ of the octagonal thin-walled columns so as to maximize the *SEA* when impact occurs on such columns.

In our problem, the response of the thin-walled box beam is $SEA(a, t)$, which is approximated using a series of the basic functions in a form of

$$\hat{y}(x) = SEA(a,t) = \sum_{i=1}^{n} \beta_i \varphi_i(a,t) \tag{3}$$

where $n$ represents the number of basic functions $\varphi_i(a, t)$. Polynomials are used here to construct these basic functions, because the polynomial is generally used to generate response surface (RS) models with a high accuracy.

In Eq. (3), the $\beta_i$, known as the regression coefficient, is estimated using the method of least squares. Suppose we have $m$ ($m>n$) observations (obtained from FE analysis (FEA)) for the yielded response $y_i$ ($y_1$–$y_m$) based on the $m$ sampling design points $(a, t)_i$, the least squares function can be expressed as

$$L = \sum_{i=1}^{m} \varepsilon_i^2 = \sum_{i=1}^{m} \left[ y_i - \sum_{j=1}^{n} \beta_j \varphi_j(a,t) \right]^2 \tag{4}$$

where the design points $(a, t)_i$ are selected from the specified design domain, $\varepsilon_i$ is the error between the response $y_i$ observed at these points and the RS approximation at that point. Afterwards, the coefficient vector $B = (\beta_1, \beta_2, \ldots, \beta_n)$ can be determined by $\partial L/\partial\beta = 0$, which is

$$B = (\Phi^{\text{T}}\Phi)^{-1}\Phi^{\text{T}}y \tag{5}$$

where $\Phi$ denotes the matrix consisting of basic functions evaluated at the $m$ sampling points, which is

$$\Phi = \begin{bmatrix} \varphi_1(a,t)_1 & \cdots & \varphi_n(a,t)_1 \\ \vdots & \ddots & \vdots \\ \varphi_1(a,t)_m & \cdots & \varphi_n(a,t)_m \end{bmatrix} \tag{6}$$

By substituting Eq. (5) into (3), the RS model is created and the response function $SEA(a, t)$ can be fully determined.

The accuracy of the developed RS model can be measured using several criteria. The relative error (*RE*) between the observed response at those sampling points $y(x)$ and the original response $\hat{y}(x)$ is

$$RE = [\hat{y}(x) - y(x)]/y(x) \tag{7}$$

Other two important properties in evaluating the model's accuracy are the sum of squares of the residuals ($SS_E$) and the total sum of squares ($SS_T$), which are

$$SS_E = \sum_{i=1}^{m} (y_i - \hat{y}_i)^2 \tag{8}$$

$$SS_T = \sum_{i=1}^{m} (y_i - \bar{y}_i)^2 \tag{9}$$

where $\bar{y}_i$ is the mean value of $y_i$.

The model's fitness can be evaluated based on the $F$ statistic, coefficient of multiple determination $R^2$, adjusted $R^2$ statistic, and root mean square error (*RMSE*), respectively, which are calculated as

$$F = \frac{(SS_T - SS_E)/n}{SS_E/(m-n-1)} \tag{10}$$

$$R^2 = 1 - \frac{SS_E}{SS_T} \tag{11}$$

$$R^2_{adj} = 1 - \frac{m-1}{m-n}(1 - R^2) \tag{12}$$

$$RMSE = \sqrt{\frac{SS_E}{m-n-1}} \tag{13}$$

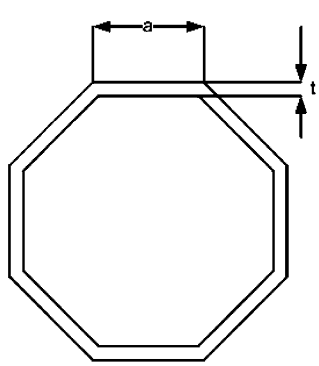


**Fig. 1.** Typical octagonal cross-section.