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Imperfection sensitivity and postbuckling analysis of elastic shells of revolution

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ABSTRACT

The imperfection sensitivity of thin shells of revolution has been a topic of great interest to researchers and designers. With the gradual emergence of the new approach of structural design based on advanced numerical simulation, the efficient and accurate prediction of the behaviour of imperfect shells has recently assumed new significance. This paper first presents an efficient semi-analytical finite element formulation for the nonlinear analysis of imperfect shells of revolution subject to general nonsymmetric loads. Both the applied loads and the initial geometric imperfections may take any form and are approximated by Fourier series expansions. Application of the analysis to study the effects of geometric imperfections on the behaviour of shells of revolution is then presented to demonstrate the accuracy and capability of the present method and the imperfection sensitivity of shell structures. As a special case of the present formulation, two different approaches are also presented for the postbuckling analysis of perfect thin shells of revolution under axisymmetric loads. The accuracy and capability of both methods are demonstrated using a number of numerical examples, which also allows some new insight to be gained into the postbuckling behaviour of perfect shells of revolution.

1. Introduction

Small geometric imperfections (or simply referred to as imperfections for brevity) are widely known to have a strong detrimental effect on the buckling strengths of shells, and to be the reason for the large discrepancy between experimental buckling loads and theoretical predictions based on the perfect geometry [1]. Since the geometric imperfection in a shell to be constructed is not known at the design stage, both the form and the amplitude of the expected imperfection have to be specified based on certain assumptions or prior knowledge in the stability assessment at the design stage. For example, the European standard for steel shell structures [2] requires that when a geometrically nonlinear elastic or elastic–plastic analysis with explicit representation of imperfections is used in design, a range of potentially damaging imperfection forms should be explored if the most unfavourable imperfection form cannot be readily identified. This code also recommends that the imperfection should be specified in the form of the critical buckling mode from a linear elastic bifurcation analysis, with its amplitude linked to fabrication quality, unless a different unfavourable pattern is justified. Such an imperfection is referred to as an eigenmode-affine imperfection in Ref. [2]. In the present study, an eigenmode-affine imperfection is more loosely defined as an imperfection in the form of an eigenmode/eigenvector from either a linear or nonlinear bifurcation analysis, which may or may not be the critical buckling mode. Only eigenmodes from a nonlinear bifurcation analysis are considered in the present paper.

The need to investigate the effect of a range of potentially damaging geometric imperfection forms in the structure to be constructed at the design stage compromises the attractiveness of the direct use of nonlinear analysis in stability design of shell structures. With today's computers, the problem often does not lie in the amount of computational time, but in the time required to prepare the finite element model and to interpret the numerical results. As the direct use of nonlinear analysis becomes widely accepted in practical design, an efficient nonlinear analysis computer program for use in shell stability design, minimizing the total user effort, is highly desirable.

Given the above considerations and noting that most civil engineering metal shell structures are axisymmetric, it is clear that the semi-analytical method for shells of revolution [3,4] using an axisymmetric shell element and Fourier series approximations of circumferential variables provides an attractive alternative to a general finite element method employing general shell elements. In the semi-analytical method, as discretization of the shell is required only in the meridional direction, the preparation of input data and the interpretation of numerical

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results are both more efficient and simpler. The basic features of such an analysis are also easier for the user to master than a general shell element analysis. In addition, it is more accurate in general, as the circumferential description of displacements in many buckling and postbuckling problems is exact. In particular, for shells of revolution subjected to axisymmetric loads and with an eigenmode-affine imperfection described by a single harmonic term, the semi-analytical method can be much more efficient than the general finite element method, as the circumferential variations of geometric imperfections and other variables can be described using a small number of Fourier terms.

Against the above background, this paper first presents a new efficient semi-analytical finite element formulation for the nonlinear analysis of imperfect shells of revolution subject to general nonsymmetric loads. The present formulation represents an extension of the formulation for nonlinear analysis for perfect thin shells of revolution under arbitrary loads presented by Hong and Teng [4]. The advantages of Hong and Teng's semi-analytical method for shells of revolution over previous formulations are detailed in Ref. [4]. Both the applied loads and the initial geometric imperfections may take any form and are approximated by Fourier series expansions. Application of the analysis to study the effects of eigenmode-affine geometric imperfections and multi-mode imperfections on the behaviour of shells of revolution subjected to axisymmetric loads is then presented. Finally, a cylindrical shell with a multi-mode imperfection under wind pressure is studied to demonstrate the greater capability of the present formulation than is illustrated by the simpler numerical examples. As a special case of the present formulation, two different approaches are presented for the postbuckling analysis of perfect thin shells of revolution under axisymmetric loads. The accuracy and capability of both methods are demonstrated using a number of numerical examples, which also allow some new insight to be gained into the postbuckling behaviour of perfect shells of revolution.

2. Nonlinear analysis of imperfect shells

2.1. Displacements of imperfect shells

The isoparametric doubly curved shell element used in the present finite element formulation is shown in Fig. 1. The accuracy of the element has been demonstrated in many successful applications [4–6]. The element geometry is described in cylindrical coordinates and is uniquely defined by the radius R, the axial coordinate z and the element meridional curvature $\mathrm{d}\varphi/\mathrm{d}s$ at the nodal points. The intermediate values of R, z, and $\mathrm{d}\varphi/\mathrm{d}s$ of the shell element are interpolated in terms of the nodal values using cubic Hermitian functions. The nodal displacements in global coordinates are taken as u_i, $(\mathrm{d}u/\mathrm{d}s)_i$, v_i, $(\mathrm{d}v/\mathrm{d}s)_i$, w_i and $(\mathrm{d}w/\mathrm{d}s)_i$ at the two nodes of the element (Fig. 1d). The displacements at any point, defined in the global coordinate system, u, v and w (Fig. 1c) are interpolated between the nodal points in terms of the nodal values also using cubic Hermitian functions. The set of global displacements u, v and w at any point is related by a transformation matrix $[T]$ to the local displacements $\bar{u}$, $\bar{v}$ and $\bar{w}$ (in curvilinear coordinates) [5].

For a perfect shell of revolution subject to arbitrary loads, the displacements in general include both symmetric and anti-

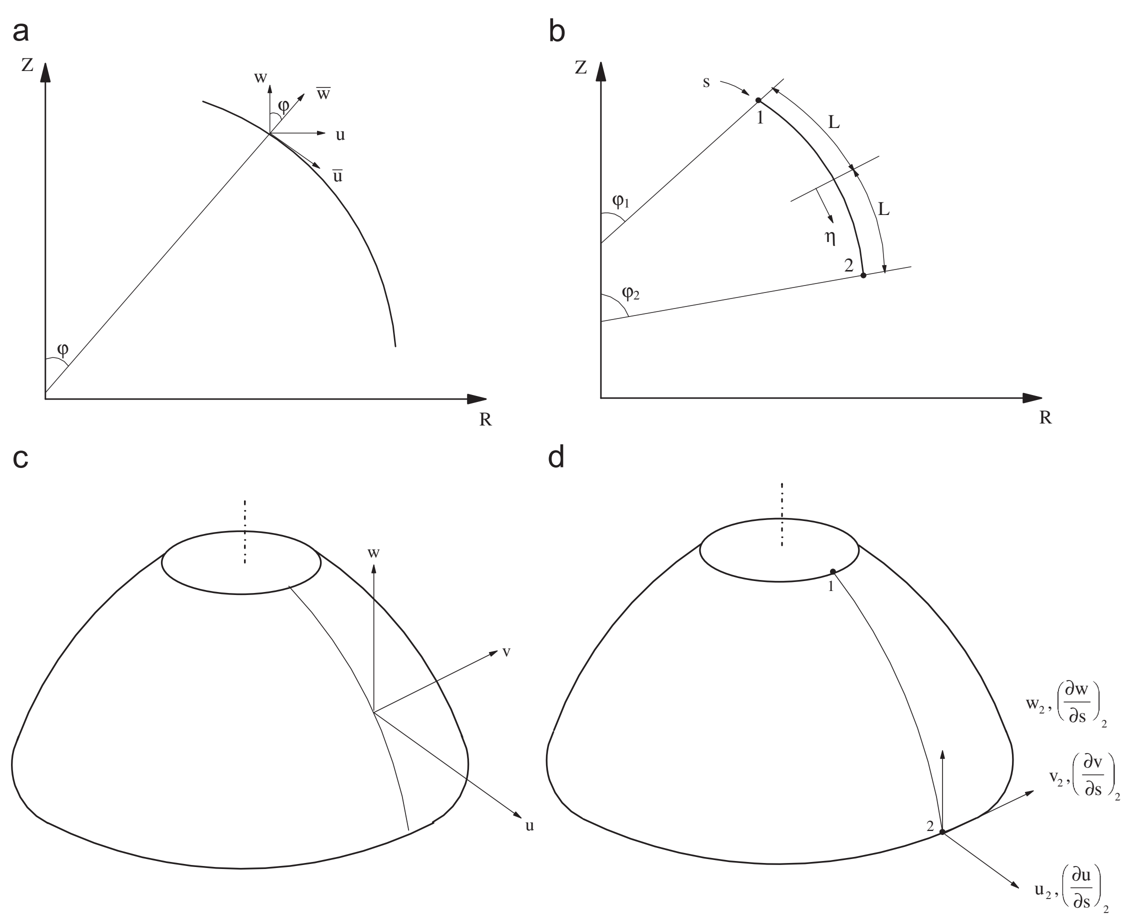

Fig. 1. Doubly curved axisymmetric shell element: (a) local and global displacements, (b) geometry of an element, (c) displacements within the element, (d) nodal displacements.