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An analytical study on the free vibration of smart circular thin FGM plate based on classical plate theory

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ABSTRACT

Analytical investigation of the free vibration behavior of thin circular functionally graded (FG) plates integrated with two uniformly distributed actuator layers made of piezoelectric (PZT4) material based on the classical plate theory (CPT) is presented in this paper. The material properties of the FG substrate plate are assumed to be graded in the thickness direction according to the power-law distribution in terms of the volume fractions of the constituents and the distribution of electric potential field along the thickness direction of piezoelectric layers is simulated by a quadratic function. The differential equations of motion are solved analytically for clamped edge boundary condition of the plate. The detailed mathematical derivations are presented and numerical investigations are performed while the emphasis is placed on investigating the effect of varying the gradient index of FG plate on the free vibration characteristics of the structure. The results are verified by those obtained from three-dimensional finite element analyses.

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1. Introduction

A new class of materials known as 'functionally graded materials' (FGMs) has emerged recently, in which the material properties are graded but continuous particularly along the thickness direction. In an effort to develop the super heat resistant materials, Koizumi [1] first proposed the concept of FGM. These materials are microscopically heterogeneous and are typically made from isotropic components, such as metals and ceramics.

In the quest for developing lightweight high-performing flexible structures, a concept emerged to develop structures with self-controlling and self-monitoring capabilities. Expediently, these capabilities of a structure were achieved by exploiting the converse and direct piezoelectric effects of the piezoelectric materials as distributed actuators or sensors, which are mounted or embedded in the structure [2,3]. Such structures having built-in mechanisms are customarily known as 'smart structures'. The concept of developing smart structures has been extensively used for active control of flexible structures during the past decade [4].

Recently, considerable interest has also been focused on investigating the performance of FG plates integrated with piezoelectric actuators. For example, Ootao and Tanigawa [5] theoretically investigated the simply supported FG plate integrated with a piezoelectric plate subjected to transient thermal loading. A 3D solution for FG plates coupled with a piezoelectric actuator layer was proposed by Reddy and Cheng [6] using transfer matrix and asymptotic expansion techniques. Wang and Noda [7] analyzed a smart FG composite structure composed of a layer of metal, a layer of piezoelectric and a FG layer in between, while in Ref. [8] a finite element model was developed for studying the shape and vibration control of FG plates integrated with piezoelectric sensors and actuators. Yang et al. [9] investigated the nonlinear thermo-electro-mechanical bending response of FG rectangular plates covered with monolithic piezoelectric actuator layers; most recently, Huang and Shen [10] investigated the dynamics of an FG plate coupled with two monolithic piezoelectric layers undergoing nonlinear vibrations in thermal environments. All the aforementioned studies focused on the rectangular-shaped plate structures.

However, to the authors' best knowledge, no researches dealing with the free vibration characteristics of the circular FGM plate integrated with the piezoelectric layers have been reported. Therefore, in conjuction with the author's recent works [11,12], the present work attempts to solve the problem of providing an analytical solution for free vibration of thin circular FG plates with two full size surface-bonded piezoelectric layers on the top and the bottom of the FG plate. The formulations are based on CPT. A consistent formulation that satisfies the Maxwell static electricity equation is presented so that the full coupling effect of the piezoelectric layer on the dynamic characteristics of the circular FGM plate can be estimated based on the free vibration results. The physical and mechanical properties of the FG

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substrate plate are assumed to be graded continuously in the thickness direction according to the power-law distribution in terms of the volume fractions of the constituents. The differential equations of motion are solved analytically for clamped edge boundary condition of the plate. By using of some mathematical techniques, these differential equations are transformed to a sixth order ordinary differential equation and finally by implementing the operator decomposition method on this equation, three Bessel types of equations are obtained which can easily be solved for the plate deflection and the potential function. The detailed mathematical derivations are presented. In numerical investigations, the emphasis is placed on investigating the effect of varying the gradient index of FG plate on the free vibration characteristics of the structure. The results are verified by those obtained from 3D finite element analyses.

2. Functionally graded materials

In an FG material made of ceramic and metal mixture, if the volume fraction of the ceramic part is represented by V_c and the metallic part by V_m, we have

$$V_m + V_c = 1. \tag{1}$$

Based on the power law distribution [13], the variation of V_c vs. thickness coordinate (z) placed at the middle of thickness, can be expressed as:

$$V_c = \left(\frac{z}{2h_f} + \frac{1}{2}\right)^g, \quad g \geqslant 0. \tag{2}$$

We assume that the inhomogeneous material properties, such as the modulus of elasticity E and the density ρ change in the thickness direction z based on Voigt's rule over the whole range of the volume fraction [14]; while the Poisson's ratio υ is assumed to be constant in the thickness direction as:

$$E(z) = (E_c - E_m)V_c(z) + E_m, \tag{3a}$$

$$\rho(z) = (\rho_c - \rho_m)V_c(z) + \rho_m, \tag{3b}$$

where subscripts m and c refer to the metal and ceramic constituents, respectively. After substituting V_c from Eq. (2) into Eq. (3), material properties of the FGM plate are determined in the power law form which are the same as those proposed by Reddy and Praveen [13] i.e.:

$$E(z) = (E_c - E_m)\left(\frac{z}{2h_f} + \frac{1}{2}\right)^g + E_m, \tag{4a}$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{z}{2h_f} + \frac{1}{2}\right)^g + \rho_m, \tag{4b}$$

$$v(z) = v. \tag{4c}$$

3. Piezoelectric materials

For symmetry piezoelectric materials in polar coordinate, the stress–strain–electric field intensity relations based on well-known assumptions of classical plate theory (CPT), can be written as [15]

$$\sigma_{rr}^{p} = \bar{C}_{11}^{E}\varepsilon_{rr} + \bar{C}_{12}^{E}\varepsilon_{\theta\theta} - \bar{e}_{31}E_z, \tag{5}$$

$$\sigma_{\theta\theta}^{p} = \bar{C}_{12}^{E}\varepsilon_{rr} + \bar{C}_{11}^{E}\varepsilon_{\theta\theta} - \bar{e}_{31}E_z, \tag{6}$$

$$\tau_{r\theta}^{p} = \left(\bar{C}_{11}^{E} - \bar{C}_{12}^{E}\right)\varepsilon_{r\theta} = -z\left(\bar{C}_{11}^{E} - \bar{C}_{12}^{E}\right), \tag{7}$$

in which σ_i, ε_k and e represent the stress and strain components and the permeability constant of piezoelectric material and E_k indicates the components of the electric field and $\bar{C}_{ij}^{E}$ are the components of the symmetric piezoelectric stiffness matrix and $\bar{e}_{31}$ is the reduced permeability constant of piezoelectric material as [16]

$$\bar{C}_{11}^{E} = C_{11}^{E} - \frac{(C_{13}^{E})^2}{C_{33}^{E}}, \quad \bar{C}_{12}^{E} = C_{12}^{E} - \frac{(C_{13}^{E})^2}{C_{33}^{E}},$$
$$\bar{e}_{31}^{E} = e_{31} - \frac{C_{13}^{E}e_{33}}{C_{33}^{E}}.$$

4. Constitutive relations

The cross section of a circular FGM plate with a piezoelectric layer mounted on its surface is shown in Fig. 1. In most practical applications, the ratio of the radius to the thickness of the plate is more than 10, and the Kirchhoff assumption for thin plates is applicable, whereby the shear deformation and rotary inertia can be omitted. For such a structure, the displacement field is assumed as follows:

$$u_z = u_z(r,\theta,t) = w(r,\theta,t), \tag{8}$$

$$u_r = u_r(r,\theta,t) = -z\frac{\partial u_z}{\partial r}, \tag{9}$$

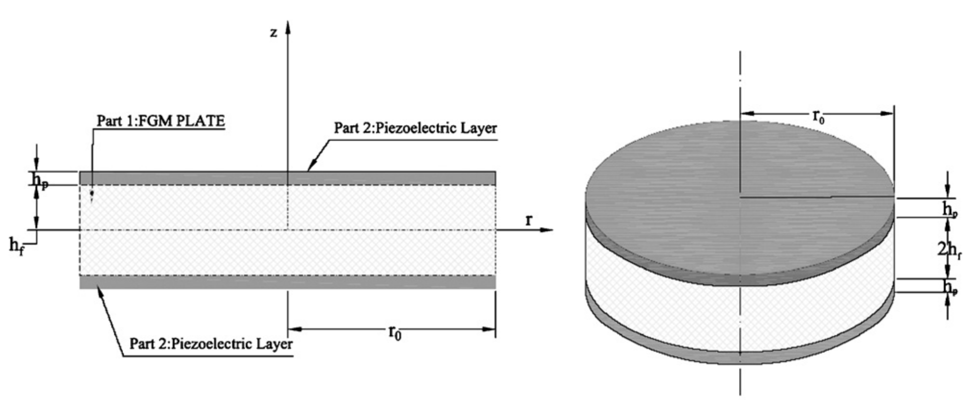

Fig. 1. Schematic representation of the FGM circular plate with two piezoelectric layers mounted on its upper and lower surfaces.

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