Thin-Walled Structures 47 (2009) 912–918

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Simplification of the derivation of influence coefficients for symmetric frusta of shells of revolution

Alphose Zingoni *

Department of Civil Engineering, University of Cape Town, Rondebosch 7701, Cape Town, South Africa

ARTICLE INFO

Article history:
Received 12 September 2008
Accepted 22 February 2009
Available online 1 April 2009

Keywords:
Shell of revolution
Shell frustum
Shell analysis
Edge effects
Discontinuity effects
Influence coefficient
Spherical shell
Symmetric shell frustum

ABSTRACT

Shell-of-revolution frusta that possess symmetry in a plane perpendicular to the axis of revolution of the shell are often encountered as parts of bigger shell assemblies, and these frusta can have a wide variety of possible midsurface geometries such as spherical, ellipsoidal, toroidal, parabolic or hyperbolic. This paper presents a new technique for the simplification of the derivation of the influence coefficients for symmetric frusta of shells of revolution. The key strategy is the reduction of the number of unknowns of the problem by decomposing a system of arbitrary shell-edge actions into symmetric and anti-symmetric components conforming to the equatorial symmetry of the configuration.

1. Introduction

Shell-of-revolution frusta that possess symmetry in a plane perpendicular to the axis of revolution of the shell are often encountered as parts of bigger shell assemblies. We will refer to the plane of symmetry that is perpendicular to the axis of revolution of the shell as the "equatorial plane". Such symmetric shell frusta can have a variety of possible midsurface geometries such as spherical, ellipsoidal, toroidal, parabolic or hyperbolic. Some examples are illustrated in Fig. 1, where the axis of revolution of the shell is denoted by $R-R$ and the equatorial plane of symmetry is denoted by $E-E$. Here the shell frustum may represent the thickened part of an elevated liquid-containment shell in the zones around the supports (which are usually located at the equator for such tanks), or the edges of the frustum may simply be junctions of the frustum to another shell of different geometry (i.e. discontinuities in slope or radius of curvature of the shell meridian).

In analytical treatments of the axisymmetric bending of shells of revolution, a flexibility-type approach is often employed, where the membrane solution is taken as an approximate particular integral of the full bending-theory equations, while a system of axisymmetric bending moments and shearing forces applied upon the shell edges is taken as the homogeneous solution [1]. The latter will be referred to as "edge actions", and their effect upon the shell as the "edge effect". This approximation is known to be very accurate in the case of thin shells of radius-to-thickness ratio r/t greater than 30 (many shells in civil and mechanical engineering belong to this category), with errors being of the order of only t^2/r^2in comparison with unity [2,3]. This allows the stresses and deformations in the entire shell to be determined by superimposing the effects of the membrane solution with those of the edge actions.

In the flexibility analysis of thin axisymmetrically loaded shells of revolution, the edge actions are initially regarded as unknowns (or "redundants" in the terminology of the *force method* of structural analysis), and appropriate compatibility conditions must be imposed at the shell edges in order to allow the evaluation of these redundants [1]. In this process, we require the values of edge rotations and displacements associated with the surface loading (these are readily given by the membrane solution), as well as edge rotations and displacements associated with an arbitrary set of edge actions.

Fig. 2(a) shows an arbitrary set of axisymmetric edge actions applied upon the upper and lower edges of a symmetric shell-of-revolution frustum: $\{M_1, H_1\}$ at the upper edge, and $\{M_2, H_2\}$ at the lower edge. The actions $\{M_1, M_2\}$ are bending moments per unit length of the respective edge of the shell, while $\{H_1, H_2\}$ represent horizontal shearing forces per unit length of the shell edge (assuming the axis of revolution of the shell $R-R$ is vertical). Fig. 2(b) shows deformations arising at the shell edges as a result of the applied edge actions: $\{V_1, \delta_1\}$ at the upper edge, and $\{V_2, \delta_2\}$ at the lower edge. The deformations $\{V_1, V_2\}$ are rotations of the shell meridian (taken as positive when anticlockwise on the left of the axis of revolution of the shell), while $\{\delta_1, \delta_2\}$ are horizontal displacements of the shell (taken as positive when away from the axis of revolution of the shell).

* Tel.: +27 21 650 2601; fax: +27 21 689 7471.
E-mail address: alphose.zingoni@uct.ac.za

0263-8231/$ - see front matter
doi:10.1016/j.tws.2009.02.005

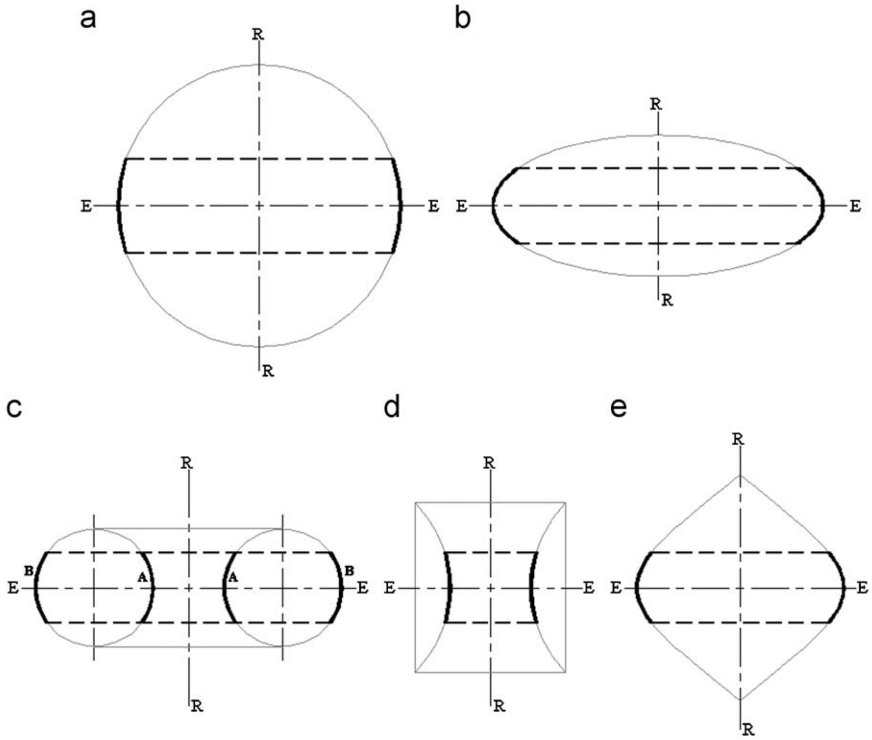

Fig. 1. Symmetric shell-of-revolution frusta: (a) spherical; (b) ellipsoidal; (c) toroidal; (d) hyperboloidal; (e) paraboloidal.

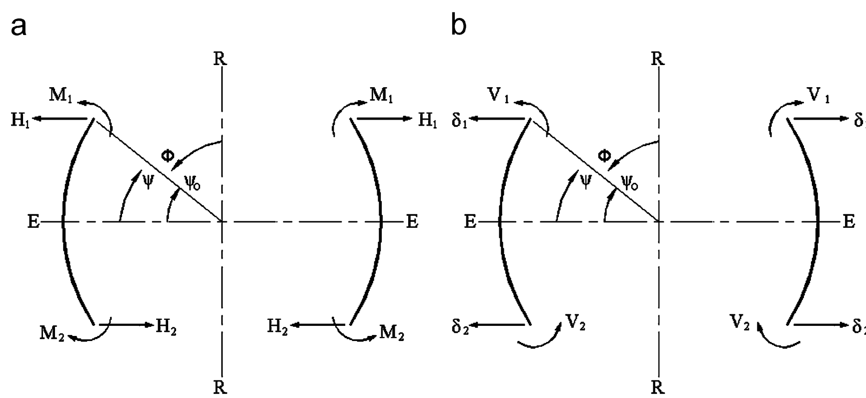

Fig. 2. Actions and deformations at the edges of a symmetric shell-of-revolution frustum (axis of revolution assumed to be vertical): (a) bending moments $\{M_1, M_2\}$ and horizontal shearing forces $\{H_1, H_2\}$; (b) rotations $\{V_1, V_2\}$ and horizontal displacements $\{\delta_1, \delta_2\}$.

We will adopt the coordinate ϕ to denote the meridional angle measured from the upward direction of the axis of revolution of the shell, to the normal to the shell midsurface at the point in question (Fig. 2). The alternative coordinate ψ denotes the meridional angle measured from the equatorial plane E–E to the normal to the shell midsurface at the point in question. From the diagrams, it is clear that the relationship $\psi = (\pi/2)-\phi$ holds. The parameter ψ_o is simply the value of ψ corresponding to the upper edge of the shell.

We may express the bending-related edge deformations in terms of the edge actions causing them in the following manner [1]:

$$\begin{bmatrix} V_1 \\ \delta_1 \\ V_2 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} I_{11} & I_{12} & I_{13} & I_{14} \\ I_{21} & I_{22} & I_{23} & I_{24} \\ I_{31} & I_{32} & I_{33} & I_{34} \\ I_{41} & I_{42} & I_{43} & I_{44} \end{bmatrix} \begin{bmatrix} M_1 \\ H_1 \\ M_2 \\ H_2 \end{bmatrix} \tag{1}$$

The I_{ij} $(i = 1, \ldots, 4;\ j = 1, \ldots, 4)$ are the influence coefficients which, if known, enable the edge deformations caused by any system of edge actions $\{M_1, H_1, M_2, H_2\}$ to be fully evaluated. The determination of influence coefficients for various shells has received much attention in the past. For instance, Stern and Tsui [4] have obtained such coefficients for thin spherical-shell frusta on the basis of a practically exact solution for the axisymmetric bending of the spherical shell, whereas Zingoni and Pavlovic [5] have exploited an approximate but accurate solution for the axisymmetric bending of non-shallow spherical shells to derive influence coefficients for spherical-shell frusta. Both studies [4,5] also establish criteria that enable the bending effects at one edge of the spherical-shell frustum to be decoupled from those at the other edge, permitting a simplification of the shell analysis. However, the determination of influence coefficients for shells of revolution of more complex geometry remains a computationally challenging task, even if an exact mathematical solution for the shell-bending problem is known for the shell geometry in