Thin-Walled Structures 47 (2009) 931–941

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Ultimate capacity of axially loaded thin-walled tapered columns with doubly symmetric sections

A.H. Salem [a], M. El Aghoury [a], M.N. Fayed [a], I.M. El Aghoury [b,*]

[a] Department of Structural Engineering, Ain Shams University, 1 El-Sarayat Street, Abbassia, P.O. Box 11517,Cairo, Egypt
[b] Department of Building, Civil & Environmental Engineering, Concordia University, Montreal, Quebec, Canada H3G 2W1

ARTICLE INFO

Article history:
Received 2 January 2009
Received in revised form
29 January 2009
Accepted 12 February 2009
Available online 8 April 2009

Keywords:
Structural engineering
Steel
Stability
Slender sections
Tapered members
Local imperfection
Global buckling
Ultimate strength

ABSTRACT

The behaviour of thin-walled I-section tapered members is governed by the width–thickness ratio of the plate elements along with the member slenderness and tapering ratios. The effect of this interaction has been studied theoretically in this work. A nonlinear finite element model which allows for geometric non-linearities is constructed for the study. Large number of members having different values of flange and web width–thickness ratios along with different member lengths and tapering ratios were selected to draw complete ultimate strength–slenderness ratio curves and to study the different modes of failure. A series of curves were drawn for design purpose. Finally, an empirical equation to get the ultimate axial capacity of tapered slender I-section members (using the whole section) is presented. The equation alleviates the current complexity in the calculation of the effective width, providing more flexible design procedure.

1. Introduction

Current trends in steel construction are to use tapered sections to minimize as far as possible the use of excess material, by choosing the cross-sections to be as economic as possible leaving by that the classical approach of using prismatic members. Also, the use of thin-walled sections has been the point of interest of many researchers in the last 50 years. While there are clear advantages to be achieved in terms of lighter weight, the combination of tapered sections and high plate slenderness leads potentially to local instability with reduced section capacities in compression and bending. Current codes of practice use the effective width concept for studying thin-walled sections, where parts of the sections that exhibit local buckling are eliminated from the cross section. Also, few codes deal with tapered members but is limited to compact and non-compact sections. Furthermore, current design interaction curves for members in combined compression and bending do not take into account the combined effect of both tapering of the beam-column and the small thickness of its section walls.

The authors will briefly classify previous research approaches considering thin-walled columns, beams and, beam-columns as well as tapered beam-columns. Generally, in thin-walled sections, formulae used to calculate the effective width are derived from the local buckling expressions of individual plates; this is expressed by the local buckling coefficient K and the maximum compressive stress F_n at the edges of plates. The drawback of this method is that it neglects the interaction between the plate elements forming the cross section when determining the coefficient. Bulson [1] and Allen and Bulson [2] have made several efforts to improve the plate bucking coefficient to account for this interaction in case of I-sections. Furthermore, they neglect the interaction between the width–thickness ratios when determining the actual stress distribution across the section which affects the value of the maximum compressive stress F_n. In 1981, Hancock [3,4] extended the finite strip method to include the nonlinear response of imperfect plate strips under longitudinal compression. He calculated the effective flexural resistance of box and I-sections under axial loads. He found that the effective section method for predicting the interaction of local and overall buckling is accurate for I-sectioned columns with very heavy webs, but over-conservative for normal web geometry. Hancock proposed a simple design method based on the Winter's [5] effective width formula. In 2002, Schafer [6], introduced the direct strength method which determines the reduced strength of cold-formed columns or beams such as channels, zed, rack, and hat sections. The method uses the gross properties of the member to determine its elastic buckling behaviour in the different modes (local,

* Corresponding author.
E-mail address: i_elagho@bcee.concordia.ca (I.M. El Aghoury).

0263-8231/$ - see front matter © 2009 Published by Elsevier Ltd.
doi:10.1016/j.tws.2009.02.011

distortional, Euler). He used that information in a series of ultimate strength curves to predicate the ultimate strength of cold-formed columns/beams. In 2004, Salem et al. [7], worked on establishing series of interaction design curves for slender I-sections in compression using the gross properties and taking into account the interaction of the width–thickness ratios of the plate elements comprising the cross sections.

The need for steel members with non-uniform cross-sections was used with the aim to minimize the total weight and subsequently the cost of the structure. Many research attempts were made to find out the behaviour of tapered members. Lee et al. [8] and Boley [9] have established that for small tapering angles (15° or less) the Bernoulli–Euler theory for beams yields satisfactory results. This has resulted in simplified analyses that have permitted the extension of design formulas for prismatic members to tapered ones. Along with the analytical studies, two test programs were carried out in recent years. The first experimental program was conducted at Columbia University under the direction of Butler [10]. In the Columbia test program, tapered I-shaped beams and channel sections tapered in both the web and flanges were tested as cantilever beam-columns. The primary interest was the elastic stability of these beams and their bracing requirements. The second experimental program under the technical guidance of the SSRC–WRC joint task committee began in 1966 by Lee and Ketter at the State University of New York at Buffalo. The results of this series of tests were reported by Prawel et al. [11]. The primary interest in the second set of tests was the inelastic stability of tapered I-shaped beam-columns. Also in this experimental study residual stresses in welded tapered shapes were measured. The magnitude and distribution of the residual stresses are very similar to those in welded built-up prismatic members.

The aim of this work is to study the effect of the interaction between flange and web width–thickness ratios along with member slenderness and tapering ratios on the behaviour of I-section columns fabricated from thin plates. Large number of members having different values of flange and web width–thickness ratios along with different member lengths and tapering ratios where selected to draw complete ultimate strength–slenderness ratio curves as well as to study the different modes of failure.

2. Finite element model

2.1. Description of the model

A nonlinear finite element model was established using ABAQUS finite element package to determine the ultimate capacity of slender tapered compression members. The element used is “S4” which is fully integrated, general purpose, finite-membrane-strain shell element. Element type S4 is a 4-noded element having six degrees of freedom per node. The material used is mild steel with yield stress of 240 MPa. The material stress–strain curve is assumed to be elastic–perfectly plastic obeying von Mises yield criterion.

The Riks method was used to predict unstable, geometrically nonlinear collapse of a structure and can be used for speed convergence of ill-conditioned or snap-through problems that do not exhibit instability. This approach provides solutions regardless of whether the response is stable or unstable.

Sections with various flange width–thickness ratios, web width–thickness ratios and tapering ratios are considered in this study. The finite element model and the boundary conditions are as shown in Fig. 1.

2.2. Modeling of geometric imperfections

Initial geometric imperfections are divided into local geometric imperfections and global geometric imperfections. Local geometric imperfections represent the local variations in the shape of the steel plate components which form the cross section from its ideal geometry. Local imperfections can be defined as a linear superposition of buckling eigenmodes obtained from a previous eigenvalue buckling analysis problem. The global geometric imperfections represent the global geometric defects that may be found along the member length such as bending, twisting, etc.

2.2.1. Overall geometric imperfections

Overall initial imperfections are considered by modeling the member with a second-degree parabola along its whole length. The maximum amplitude of overall imperfection at the member mid-length and about the minor axis of the cross section is chosen as $L/1000$ as shown in Fig. 2.

2.2.2. Local imperfections

Hancock [3,4], suggested that the distribution of local plate imperfections may be assumed similar to the expected local buckling shape of the plate. The critical buckling shape of axially loaded member with slender I-section consists of a half-sine wave in the transverse direction and series of half-sine waves in the longitudinal direction. Several trials showed that, the numbers of longitudinal half-sine waves are approximately equal to the length of the column “L” divided by the height of the web “H_w” which is similar to the local buckling pattern of uniaxial loaded plate with length “L” and height “H_w”.

For verification purposes, a slender I-section column with length equal to 4500 mm is modeled for different initial local imperfection values, d_o. For the cases where “d_o” ranges form ($H_w/1000$) to ($H_w/250$), the ultimate axial loads are nearly equal. For the cases where “d_o” varies between ($H_w/100$) and ($H_w/50$), the ultimate axial load decreases gradually, which means that the effect of local imperfection is clear and dominates the behaviour. The maximum initial imperfection value used by Hancock was 10% of the plate thickness. Sivakumaran et al. [12] said that the British steel design code suggests the use of the following formula as an upper limit for the imperfection amplitude, “do”, of compression steel plates which is:

$$\frac{d_o}{t} = 0.145\left(\frac{w}{t}\right)\sqrt{\frac{F_y}{E}} \quad (1)$$

where “t” is the thickness of the plate element, “w/t” is the slenderness ratio, “E” and “F_y” are the material properties of the plate. From the previous studies, the authors used an average “d_o” within the practical limits which is equal to ($H_2/400$). For tapered sections, all buckling analysis results showed that the most probable section where web buckling is initiated is near the bigger end named H_2 (Refer to Section 3).

2.2.3. Modeling local and global imperfections in the finite element model

In the first analysis run, an eigenvalue buckling analysis is performed with (ABAQUS/Standard) on the “perfect” structure to establish probable collapse modes and to verify that the mesh discretizes those modes accurately. The perfect structure in our case will be the one with overall imperfection already introduced, see Figs. 3 and 4. Then the eigenmodes will be written in the default global system to the results file as nodal data. The lowest buckling modes are frequently assumed to provide the most critical imperfections, so usually these are scaled and added to the perfect geometry to create the perturbed mesh. The imperfection