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Multi-parameter numerical optimization of selected thin-walled machine elements using a stigmergic optimization algorithm

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Abstract

This paper presents a method of multi-parameter optimization of shell structures of constant thickness from linearly elastic homogeneous material. Our goal is to present how a relatively complex shape of shell can be optimized in terms of its stiffness only by changing its geometry and at the same time preserve the primary volume. Loading cases are not known exactly and so we use the criterion function which on the basis of arbitrary chosen eigenvalue frequencies, gives a stiffness of the shell structure. The optimization algorithm is based on stigmergy, which is becoming increasingly popular in combinatorial optimization and lately also in multi-parameter optimization. The method was verified on a real industrial problem on an electric motor casing.

Keywords: Thin-shell structure; Stiffness; Stigmergy; Optimization; Multilevel approach

1. Introduction

In the design of various thin-shell structures we often encounter the problem of modeling a shell-structure as light as possible and, at the same time satisfying the requirements of high stiffness. To achieve this, one possibility is that at the proper places on the shell material is added, thus reinforcing the shell, structure. However, in this way we increase the time of manufacturing, amount of material and difficulty of processing. Another possibility is trying to make the shell stronger only through variation of its shape.

Our starting condition in this optimization study was that the volume of the newly shaped structure has to remain the same as in the existing one, that the consumption of material is not higher and that the manufacturing process and time are not affected by the change in the design. This kind of approach is especially important for cases of low-cost components which are mass-produced by deep drawing. In the continuation of this paper, an example of such optimization will be presented in detail on the casing of the electric motor produced by Domel Ltd., Železniki, Slovenia.

2. Optimization criterion

The optimization criterion of a parametrically-defined object is a function which estimates this object with a certain numerical value. Formally, a multi-parameter numerical optimization problem $N = (P, D, \Omega, f, S, extr)$ is defined by:

- a set of parameters $P = p_1, \ldots, p_n$, also called a solution space,
- a set of continuous domains $D = D_1, \ldots, D_n$ of parameters, where the parameter p_i is taken from the continuous domain D_i, $i = 1, \ldots, n$,
- a finite set of constraints, Ω, defined over the parameters P,
- a criterion function $f_c : D_1 \times \cdots \times D_n \longrightarrow \mathbb{R}$,
- a set of feasible solutions $S = \{s = (p_1, \ldots, p_n) | \ p_i \in D_i \wedge p_i$ satisfies constraints, Ω, for $i = 1, \ldots, n\}$,
- the extreme $extr$, which is min or max.

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Usually, S is called a space of feasible solutions. Each element of a set S is a candidate for a final solution. To solve a numerical optimization problem, a solution $s^* \in S$ has to be found such that an *extr* value is returned by the criterion function f_c. Therefore, $f_c(s^*) \leqslant f_c(s)$ must be true for $\forall s \in S$ in the case when we are looking for a minimum, or $f_c(s^*) \geqslant f_c(s)$ must be true $\forall s \in S$ in the case of a search for a maximum. Solution s^* is called a global optimum solution of the problem N. The set of global optimum solutions for a given problem is marked by S^*.

The set of parameters $P = p_1, \ldots, p_n$ gives us certain segments of the structure which can be geometrically varied within the constraints domain Ω. So parameter p_i can be either a radius of some rounded region, radius of a bore, slope of surface, or magnitude of protrusion in the defined region, etc. Thus with parameters we cause the changes in the basic geometry of the structure which is then estimated by criterion function in the way that we interpret geometry as a mechanical part influenced by the surrounding.

The criterion function is relatively simply defined when loadings are deterministic. Usually we optimize such cases with the minimization of displacements or stresses, maximization of elasticity, etc. [1–3]. It is difficult to determine the criterion function for a dynamically loaded assembly where the loads are stochastic. So the question is how to define the stiffness of a shell structure if we do not know the lateral loads.

For this purpose we shall define the kinetic or internal energy T and deformation work or work of external forces W. If we consider an arbitrary linearly elastic body of volume V and density ρ placed in an orthogonal coordinate system, it follows [4]:

$$T = \frac{1}{2}\int_V \rho v_i v_i \,\mathrm{d}V = \frac{1}{2}\int_V \rho \dot{u}_i \dot{u}_i \,\mathrm{d}V \tag{1}$$

and

$$W = \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij}\,\mathrm{d}V. \tag{2}$$

In Eq. (1) $v_i = \dot{u}_i = (\dot{u}, \dot{v}, \dot{w})$ is a velocity vector where u, v and w are displacements in a particular coordinate axis and their derivatives with respect to time are denoted by a dot above them. Further in Eq. (2)

$$\sigma_{ij} = \begin{pmatrix} \sigma_x & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_y & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z \end{pmatrix} \tag{3}$$

and

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z \end{pmatrix}. \tag{4}$$

are the stress and strain tensor, respectively.

Generally, the components of strain tensor are the following:

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}), \tag{5}$$

but for small displacements the last component in the above equation can be neglected; $u_{k,i}u_{k,j} \approx 0$. Now the relation between the stress and strain tensor for a linearly elastic body can be written as

$$\sigma_{ij} = \frac{E}{1+v}\varepsilon_{ij} + \frac{vE}{(1+v)(1-2v)}\varepsilon_{kk}\delta_{ij}, \tag{6}$$

here E is elastic modulus of material, v is shear or *Poisson*'s ratio and δ_{ij} is the so called *Kronecker* delta tensor.

Eq. (1) can now be transformed in the following way:

$$T = \frac{1}{2}\int_V \rho\left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right)\mathrm{d}V. \tag{7}$$

Taking into account the *Rayleigh–Ritz* approximation [5], we can write the elements of displacements derived with respect to time $\dot{u}_i\dot{u}_i \approx \omega u_i u_i$, and for now let ω be defined as a scalar quantity. With this replacement and assuming that the density of material ρ is constant we can determine the kinetic energy T as

$$T = \frac{\rho\omega^2}{2}\int_V (u^2 + v^2 + w^2)\,\mathrm{d}V. \tag{8}$$

By inserting Eq. (6) into Eq. (2) deformation work can be written as

$$W = \frac{1}{2}\int_V \left(\frac{E}{1+v}(\varepsilon_{ij})^2 + \frac{vE}{(1+v)(1-2v)}\varepsilon_{ij}\varepsilon_{kk}\delta_{ij}\right)\mathrm{d}V. \tag{9}$$

The last expression can be described in an expanded form if the components of strain tensor are taken into account:

$$W = \frac{1}{2}\int_V \left(\frac{E}{1+v}(\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2) + \frac{2E}{1+v}(\varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{xz}^2) + \frac{vE}{(1+v)(1-2v)}(\varepsilon_x\varepsilon_{yz} + \varepsilon_y\varepsilon_{xz} + \varepsilon_z\varepsilon_{xy})\right)\mathrm{d}V, \tag{10}$$

where the symmetry of strain tensor $\varepsilon_{ij} = \varepsilon_{ji}$ was considered. The symmetry also holds for the stress tensor ($\sigma_{ij} = \sigma_{ji}$).

However, for deformable and conservative systems in equilibrium state, the potential energy Π has its extreme, which is a minimum. If the deformations are small, then the following holds:

$$\delta\Pi = \delta T - \delta W \Rightarrow T - W = 0 \tag{11}$$

and

$$\omega^2 = \frac{\int_V \sigma_{ij}\varepsilon_{ij}\,\mathrm{d}V}{\rho\int_V u_i u_i\,\mathrm{d}V}, \tag{12}$$

where the δ symbol is a differential operator.

Because in our case, besides density the elasticity of material E is constant too, we can express the criterion